

DELAY IN COMMUNICATION AND COMPUTER NETWORKS*

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I. INTRODUCTION

Delay in communication and computer networks has recently become a subject of considerable interest. In this paper we address ourselves to the topics of analysis and optimization of such nets. Those we consider are of the store-and-forward type more commonly known as message-switching networks.

The problem confronting the network designer is to create a system which provides suitable network performance at an acceptable system cost. Since, in message-switched networks the messages experience queueing delays as they pass from node to node, the performance measure is usually taken to be the speed at which messages can be delivered. The optimization problem is to achieve minimal average delay at a fixed network cost by appropriately choosing the network topology, the channel capacity assignment, and the message routing procedure. The purpose of this paper is to review some of the methods for handling various aspects of this problem.

II. ANALYTICAL TOOLS

The appropriate tools are those which have developed from queueing theory.

II.1. Single Server Systems. Much of queueing theory considers systems in which messages (customers) place demands for transmission upon a single communication channel (the single server). When the average demand for service is less than the capacity of the channel to handle these demands, the system is said to be stable. The literature on stable single server queueing systems is fairly voluminous as for example exemplified by the excellent work by Cohen [1]. Single server systems are characterized by $A(t)$, the distribution of interarrival times and $B(x)$, the distribution of service times. In the case when $A(t)$ is exponential (i.e. Poisson arrivals), then the literature contains fairly complete results. However, when both $A(t)$ and $B(x)$ are arbitrary, then the situation becomes much more complex and only weak results are available.

Recently attention has been directed to developing approximate solution methods. These methods include: placing bounds on the behavior of the system; studying the system behavior under light and heavy traffic conditions; and by forming diffusion approximations to the physical queueing systems. This last approach appears most promising and involves replacing a discrete random process with a continuous random walk typically with a reflecting barrier at the origin to prevent queue sizes and waiting times from going negative (see Gaver [2]). Numerical results which have been obtained using the diffusion approximation have been startling in terms of their accuracy when compared to the original queueing problem.

II.2. Multiple Nodes and Networks. The case of interest to this paper is that of multiple nodes in

a network environment. The queueing problems encountered in networks are far more difficult than single server problems. The difficulty arises due to new phenomena which occur in networks, the most important of which is that traffic entering a node in the network is dependent upon traffic elsewhere in the network and on other nodes through which this traffic has passed. This difficulty manifests itself in that $A(t)$ for a network node is no longer exponential. A second difficulty is the phenomenon of blocking which occurs when the finite storage capacity of a node becomes filled and the further reception of messages is temporarily prohibited. This then places a burden on neighboring nodes and they too tend to get blocked causing the effect to propagate in the network. This effect is probably one of the least understood queueing effects in the study of nets and has significant impact upon performance.

The problem in which customers are permitted to move among a collection of queueing stations in some random fashion was studied by Jackson [3]. His major result was to show, when the system is stable, that each node in the system could indeed be analyzed as a single queueing facility (under Markovian assumptions). This represents perhaps one of the first successful attempts at decomposing a network problem into a series of simpler single node problems. Another fundamental result which permits decomposition of queueing networks is due to Burke [4]; he showed that if $A(t)$ and $B(x)$ are exponential, then the departure times are also exponentially distributed. Thus, we preserve the Poisson nature of the traffic flow between network nodes.

The results referred to in the previous paragraph do not carry over trivially into message oriented communication or computer nets since messages maintain their lengths as they pass through the net. The first comprehensive treatment of communication nets was carried out by Kleinrock [5]. Fortunately, it could be shown for a wide variety of communication nets that it was possible to introduce an assumption which once again permitted a decomposition of the network into a collection of single nodes.

Using the simple structure of the linear equations of motion governing Markovian queues, Wallace [6] has developed a procedure for solving the system of equations numerically.

III. OPTIMIZATION TOOLS

Perhaps the first communications network optimization problem was posed and solved by Kleinrock [5] in which he assumed that the network topology and the channel traffic were known quantities. Also, he assumed that the traffic was Markovian (Poisson arrivals and exponential message lengths) and justified certain decomposition assumptions. For each channel the optimal assignment of capacity C_i was found which minimized the average network delay T to messages, at a fixed total system cost D . We define: T_i as the average queueing plus transmission time on

*This work was supported by the Advanced Research Projects Agency, Dept. of Defense #DARHC15-69-C-0285.

the i^{th} channel; λ_i as the average message traffic on the i^{th} channel; γ as the average network traffic throughput; and d_i as the cost factor on the i^{th} channel. This problem takes the following form:

Problem A: Choose the set of channel capacities, C_i , to minimize T at fixed cost D where

$$T = \sum_i (\lambda_i/\gamma) T_i \quad (1); \quad D = \sum_i d_i C_i \quad (2)$$

The solution to this problem assigns a capacity to the i^{th} channel in an amount equal to the average traffic carried plus an excess capacity proportional to the square root of that traffic. It may be observed that a related capacity assignment (namely, that which gives capacity directly in proportion to traffic carried) provides an average message delay not significantly worse than the optimum.

These, and other related results, were published as Kleinrock's Ph.D. thesis (MIT) in 1962 (this work later appeared as [5]). Little was published in this field from then until 1969 [7]. Whatever the reason for this inactivity, it is clear that the recent interest is due to the development of computer networks. In 1967 Roberts [8] proposed the idea of an experimental computer network which later developed into the Advanced Research Projects Agency (ARPA) computer network (recently reported upon in the 1970 SJCC Proceedings).

In a forthcoming paper by Meister et al. [9], it is observed that in minimizing T in Problem A above, certain of the channels produce rather large and undesirable message delays T_i . As a result, Meister et al. pose the following problem, whose solution is closely related to that of Problem A:

Problem B: Same as Problem A except T is given by

$$T = \left[\sum_i (\lambda_i/\gamma) T_i^k \right]^{1/k} \quad (3)$$

By raising T_i to the k^{th} power they find that for $k > 1$, one forces a reduction in the variation among the T_i . For $k \rightarrow \infty$ the minimization yields a constant value for T_i . When $k = 0$, the assignment reduces to the proportional channel capacity assignment. The amazing observation is that T increases very slowly as k grows from unity. Moreover, they show that the variance of message delay is minimized when k is chosen equal to 2.

In Ref. [7] Kleinrock introduced some first attempts at modelling computer nets and was able to show that simple models were extremely useful in predicting the behavior of the message delay in the ARPA computer net. In a subsequent paper [10] he introduced the following variation to Problem A, since the cost function as given in Eq. (2) was found not to represent tariffs for high speed telephone data channels:

Problem C: Same as Problem A except D is given by

$$D = \sum_i d_i C_i^\alpha \quad (4)$$

where $0 \leq \alpha \leq 1$. The solution to Problem C cannot be given in closed form. Nevertheless, in applying a numerical solution of this problem to the ARPA net, it was found that the message delay varied insignificantly with α for $.3 \leq \alpha \leq 1$. This indicates that the closed form solution to Problem A may serve as

an approximation to the more difficult Problem C.

IV. ADDITIONAL CONSIDERATIONS

Minimizing cost at fixed average message delay by appropriately choosing channel capacity is the dual for problems A, B, and C. This was studied in [10] and considered recently by Whitney [11]. Choice of network topology was considered in Kleinrock's original work. Recently Frank et al. [12] considered this problem for the ARPA net and developed suboptimal search procedures. They also addressed the problem of choosing an optimal channel assignment when capacities must be chosen from a finite set; Whitney [11] and Doll [13] considered this problem for a fixed tree topology. Frank et al. [14] devised an optimal procedure for selecting discrete channel capacities for centralized computer networks.

Message routing procedures must also be considered. Of all those so far discussed, this problem lends itself least to analysis. Lastly, we note that the ultimate standard in these problems is measurement of real systems. This is receiving considerable attention in the ARPA net.

V. CONCLUSION

The attempt in this paper has been to describe and to evaluate various tools for studying delay in communication and computer nets. These tools must be considerably improved. Nevertheless, they have been useful in network studies. Among the most difficult remaining problems we mention the blocking effect due to finite storage capacity, the analyses of routing procedures, and the design of network topologies.

REFERENCES

1. J. Cohen, *The Single Server Queue*, Wiley (1969).
2. D. Gaver, "Diffusion Approximations and Models for Certain Congestion Problems," *JAP*, 5, 1968.
3. J. Jackson, "Networks of Waiting Lines," *Oper. Res.*, 5, 1957.
4. P. Burke, "The Output of a Queueing System," *Oper. Res.*, 4, 1956.
5. L. Kleinrock, *Communication Nets*, McGraw-Hill (1964).
6. V. Wallace, "Representation of Markovian Systems by Network Models," *SEL TR42*, U. of Mich., 1969.
7. L. Kleinrock, "Models for Computer Networks," *Proc. ICC*, Boulder Colo., 1969.
8. L. Roberts, "Multiple Computer Networks and Inter-computer Communications," *ACM Symposium on Operating Systems Principles*, Gatlinburg, Tenn., 1967.
9. B. Meister, H. Mueller, and H. Rudin, "New Optimization Criteria for Message-Switching Networks," *IBM Zurich Res. Lab.*, Switzerland, 1970.
10. L. Kleinrock, "Analytic and Simulation Methods in Computer Network Design," *Proc. SJCC*, May 1970.
11. V. Whitney, "A Study of Optimal File Assignment and Communication Network Configuration in Remote-Access Computer Message Processing and Communication Systems," *SEL TR 48*, U. of Mich., 1970.
12. H. Frank, I. Frisch, and W. Chou, "Topological Considerations in the Design of the ARPA Computer Network," *Proc. SJCC*, May 1970.
13. D. Doll, "Efficient Allocation of Resources in Centralized Computer-Communication Network Design," *SEL TR 36*, U. of Mich., 1969.
14. H. Frank, I. Frisch, W. Chou, and R. Van Slyke, "Optimal Design of Centralized Computer Networks," *Proc. ICC*, San Francisco, 1970.