ON THE BEHAVIOR OF A VERY FAST BIDIRECTIONAL BUS NETWORK

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ABSTRACT

In this paper, we study the behavior of a very fast bidirectional bus system. The bidirectional bus has been investigated in the past under the main assumption that the propagation delay incurred by a packet is relatively small in comparison to its transmission time. Under this assumption, it has been shown that if the packet transmission time decreases, the performance of existing access schemes (like CSMA) degrades. Recent technological developments in communication networks (such as fiber optics) have made possible much faster bus networks. For these networks, it no longer may be assumed that the propagation delay is relatively small in comparison to the transmission time. This paper deals with analyzing the very fast bidirectional bus system. In contrast to previous studies, the assumption that the bus is very fast is inherently embedded in the system model. The results derived in this paper show that due to self synchronization properties observed in the system at high loads, the system performance is not necessarily poor, as implied from previous studies.

1. Introduction and Previous Work

In a local area network, a channel is shared among many stations which are (relatively) close to each other. One of the common topologies for such a network is the bidirectional bus (e.g., Ethernet), and one of the most popular access schemes for this topology is Carrier Sense Multiple Access (CSMA). In CSMA, a station senses the channel before transmitting. If the channel is idle, the station transmits right away; otherwise, it stays silent and postpones transmission for a later time. (An improvement of CSMA is CSMA with Collision Detection (CSMA-CD). In this scheme, in addition to carrier sensing, a station can listen to the channel while it is transmitting and therefore can detect if it is involved in a collision. If a collision is detected, the station aborts its transmission and repeats the scheme described above.) Both access schemes take advantage of the very short end-to-end propagation delay (relative to the transmission time). The ratio between the propagation delay and the packet transmission time is denoted by a and can be thought of as the number of packets "contained in" the bus:

$$a \triangleq \frac{\text{propagation delay}}{\text{packet transmission time}}$$

The performance of CSMA was studied by Kleinrock and Tobagi in [Klei74, Klei75, Toba74]. The performance of CSMA-CD was studied by Tobagi and Hunt [Toba79] and by Lam [Lam80]. These studies were based on the underlying assumption that the parameter a is small, so packets are heard simultaneously by all stations (i.e., $a \ll 1$). Two important properties were observed with respect to these access schemes: 1) The attained throughput, S, of both systems increases with the offered load, G, until it reaches its maximum. After this point (very high load), the throughput decreases. 2) The maximum attainable throughput, denoted by the system capacity, decreases with a. It is observed that the performance of these schemes is good as long as $a \le .05$ (capacity of about 70%). For larger values of a (like $a = 1$), the capacity of these systems may go as low as 20%.

Technological developments (such as fiber optics) in communication networks have recently increased the speed of the communication channel, and future developments are likely to increase it even further. Other technological improvements are likely to allow future local area networks to use much longer cables. These trends lead the communication industry to the building of systems where the parameter a is larger and larger. One possibility for analyzing these new systems is to follow the approach taken in [Klei74, Klei75, Toba74, Toba79, Lam80] and use the throughput/capacity expression derived there. Doing so, we soon realize that the capacity of these systems approaches zero as a increases, and thus the use of CSMA or CSMA-CD may be very inefficient for these systems.

The goal of this paper is to challenge this "discouraging" result which predicts that the throughput of CSMA on very fast networks is very close to zero. We depart from the previous studies by discarding the assumptions that a is small and that packets are instantaneously received by all stations. Instead, we use the fact that a is large as an *underlying assumption* and create a model in which the propagation process is *inherently modeled* (rather than being assumed to be instantaneous). The main feature of the adopted model is that different packets are heard at different times by different stations. This creates a discrepancy among the stations (rather than uniformity, as in the previous models), and thus causes the system to have several positive (and quite surprising) properties: 1) The capacity of the system, under deterministic and scheduled arrivals, is close to 2 (in contrast to a capacity of 1 in the previous models). 2) Under stochastic arrivals, the system is stable: an increase in the offered load leads to an increase in the throughput. 3) The system capacity, under stochastic arrivals and using the CSMA capability, is 1 (and not close to zero, as previously may

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have been predicted). Thus we conclude that, due to asymmetry between the stations, the performance of these networks is much better than what would otherwise be predicted by the fully-symmetric, "traditional" models.

The structure of this paper is as follows: in Section 2 the system model is described. In Section 3, we study the theoretical limitations of the very-fast shared bus system. The main goal in this section is to calculate the maximum throughput which can be achieved in the system, neglecting the randomized behavior of the system inputs. The capacity of the system, defined to be the highest attainable throughput, is derived in this section under several assumptions. In Section 4, we investigate the system behavior under the assumption of stochastic arrivals. The model used in this section is similar to the models used in the analysis of slotted ALOHA and CSMA; however, this model captures the correlation between events occurring in the system. The main property discovered in this analysis is that, in contrast to previously studied shared channel systems, this system is very stable and the system throughput always increases with the offered load.

Lastly, let us say a few words about recent, related work. In an independent study, Sohraby, Molle and Venetsanopoulos [Sohr84, Sohr85, Sohr86, Sohr87] studied the performance of CSMA in fast bus systems. The similarity between that study and the present analysis is in the explicit modeling of the packet propagation, and in discovering the network asymmetry which implies good performance. Their work is different from ours in some aspects of the modeling and in dealing with systems with *large a*, which is they bound to be $a < 1/2$; in contrast, we deal with *very large a* ($a = O(N)$, where N is the number of stations). The behavior of fast bus systems has also been investigated in several other studies. However, those studies concentrated on suggesting semi-organized access schemes for these networks, and not on studying the behavior of these networks under the CSMA scheme. The main principle of those access schemes is to organize the packets transmitted in the system to efficiently use the channel. Such studies are reported in [Frat81, Gerl83a, Gerl83b, Limb82].

2. Model Description

The system consists of N stations connected by a bidirectional bus and numbered $1,2,\cdots,N$ from left to right. It is assumed that the stations are located on the bus such that the distance between every two neighboring stations is exactly one distance unit. The length of a fixed size packet, measured in terms of distance units, is assumed to be smaller than or equal to the unit distance between two neighboring stations. This implies that the parameter a of this system is $a \geq N-1$. For simplicity, we assume that the packet size exactly equals the distance between neighboring stations, i.e., $a = N-1$. Time is slotted, with the slot size equal to the time required to transmit a packet. The time interval, starting at time t and ending at time $t+1$, is called the tth *slot*. Every packet transmission starts at the beginning of a slot.

Due to these assumptions, it is not sufficient to characterize the system events by their timing only; rather, a space-time characterization of events is required. We therefore represent the system behavior using a space-time domain where the horizontal axis represents the location on the bus and the vertical axis represents time (progressing down the page). The propagation of a packet is represented by a band (see Figure 1, where station 2 transmits a packet during slot t, and stations 1 and 3 hear it during slot $t+1$).

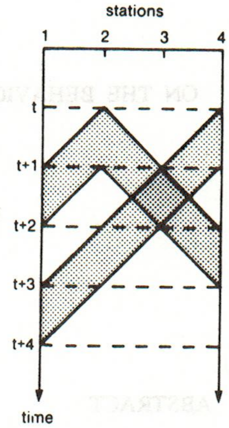

Figure 1: Two Packets Pass Through Each Other

In contrast to the traditional model, packets which collide are not assumed to destroy each other. Rather, they are assumed to "pass through" each other. For example, consider the two packets depicted in Figure 1. The packets are concurrently transmitted by stations 2 and 4 during slot t. During slot $t+1$, the packets collide at station 3 and thus neither of them is heard properly by station 3. However, the packets "pass through" each other, so during slot $t+2$, one of them is heard correctly by station 2 and the other is heard correctly by station 4. This assumption is valid, for example, when the bidirectional channel is implemented by two one-directional fiber buses.

From the above description, it is implied that the terms *idle slot*, *successful slot* and *collision slot* are not global properties of the system, but rather, are local properties of a given station. Therefore, all references to a particular slot will include both a time and location reference. For example, we say in Figure 1 that slot $t+1$ is an *idle slot* at stations 2 and 4, a *successful slot* at station 1 and a *collision slot* at station 3.

It is important in the context of this model to accurately define the notion of *successful reception*. Although the transmission medium is a *broadcast medium* (i.e., a single packet may be heard by all stations), we assume that the messages themselves are not of the broadcast type, but rather, of the point-to-point (PTP) type. This means that every packet is targeted for a single destination, and only that destination needs to receive it properly. Following this assumption, we define the *successful hearing* and *successful reception* of a packet as follows. A packet is said to be *successfully heard* by station i in slot t if the packet is heard by i in slot t, and t is a successful slot at i. A packet is said to be *successfully received* by station i in slot t if it is destined for station i and is successfully heard by i in slot t.

Politeness and Fairness

Two important properties of multiple access algorithms are to be discussed in this paper: *politeness* and *fairness*. A station is said to be *polite* if it does not transmit when it hears a transmission originated from another station. Note that politeness is a desired property which is utilized in CSMA algorithms using the Carrier Sense mechanism of the stations. Nevertheless, Carrier Sense in its traditional form will not be very effective in this slotted environment, and needs to be slightly enhanced. This issue can be best explained using Figure 1, where we consider the action taken by station 2 in slot $t+2$. In order to prevent station 2 from interfering with the currently passing packet (transmitted from station 4 during slot t), it is required that station 2 will be polite during this slot. To enforce this politeness, station 2 needs to make a decision at the very beginning of slot $t+2$ as to whether to transmit or not, based on what appears on the channel at that moment. This can be done only if the station has some "look-ahead" mechanism by which it can tell at time t^- what will be the channel status at time t. This "look-ahead" mechanism can be easily constructed by bending the bus in the neighborhood of the station in an Ω shape, and having the station tapped to the Ω leg for "look-ahead" sensing, and to the Ω head for transmission or reception.

In addition to this general politeness, we define *directional* politeness. A station is said to be *polite to the left (right)* if it does not transmit when it hears a transmission originated from a lower (higher) index station (i.e., a transmission that arrives from the left (right), according to our representation).

A transmission policy is called *fair* if for every two stations i and j, station j is allowed to transmit a packet between any two consecutive transmissions of station i. A transmission policy is called *strictly fair* if for every four stations i, j, k and l, station i is allowed to transmit to station j between any two consecutive transmissions from station k to station l.

3. On the Capacity of the System

In this section, we study the capacity of the system under various conditions. The goal is to determine the maximum system throughput which can be achieved when perfect scheduling is used. The importance in deriving this measure is to understand the system limitations, and to compare the system potential to that of other systems.

To define system throughput, recall that a packet is considered to be successful if it is heard successfully by its destination station. Let $P(t)$ be the number of packets that have been successfully received by time t; then the system *throughput*, denoted by S, is defined to be $S \triangleq \lim_{t\to\infty} P(t)/t$. The *system capacity* is defined to be the highest throughput which can be achieved by using a perfect scheduling algorithm (which can perfectly schedule the transmissions of every station).

For most communication systems, it is straightforward to derive the system capacity. For example, the capacity of a system consisting of two stations connected by a point-to-point link is 1, since at most one packet of information can be transmitted in that system per packet transmission time. Similarly, the capacity of a fully connected N-node network (where each of the links is a point-to-point link and N is even) is $N/2$, since this is the number of conversations that can be concurrently held in the system. The capacity of the bidirectional bus system, as considered above but under the assumption that the parameter a is small, is 1, since at most one station can transmit at a time.

In contrast to all these systems, the dependency of events in our system upon both time and location requires a more careful analysis of the capacity. In the following, we derive both upper and lower bounds on the system capacity.

3.1 Two Upper Bounds on the System Capacity

Before deriving the bounds, some additional notation is required. A point (i,t) in the space-time domain is called a ***transmission point*** if station i transmits a packet in slot t (i.e., starts transmitting at time t). A point (i,t) in the space-time domain is called a *reception point* if station i successfully receives a packet during slot t. A line which contains the points $(t, 1), (t+1, 2), (t+2, 3), \cdots, (t+N-1, N)$ is called a *left diagonal* (a diagonal that starts from top left and goes to bottom right). Similarly, a *right diagonal* is defined. Next, let us derive two upper bounds for the system capacity.

THEOREM 1: For any scheduling policy the system throughput obeys: $S \le N/2$.

Proof: Let $T(t)$ and $R(t)$ be, respectively, the sets of transmission points and reception points (i, t') such that $t' \le t$. Let (i, t_1) be a reception point in $R(t)$; then there exists a transmission point $(j, t_2) \in T(t)$ which uniquely corresponds to (i, t_1). This is the transmission point which corresponds to the transmission of the packet successfully received at (i, t_1). For this reason, we conclude that $|T(t)| \ge |R(t)|$. In addition, the two sets $T(t)$ and $R(t)$ must be disjoint since a station cannot transmit and receive concurrently, so the number of points in the joint set cannot exceed the number of points in the $N \times t$ rectangle, namely $|R(t)| + |T(t)| \le Nt$. Thus, from the two inequalities: $|R(t)|/t \le N/2$, and the claim follows since $|R(t)|$ is the number of packets, $P(t)$, successfully received by time t. ■

THEOREM 2: For any scheduling policy the system throughput obeys: $S \le 2$.

Proof: To prove the claim, first consider a system (called SYS1) consisting of a single *unidirectional bus*. Assuming that the transmission direction is from left to right, we examine the space-time domain and observe that on every left diagonal there may be at most one reception point. For this reason, the number of reception points in the $N \times t$ rectangle must obey $|R(t)| \le t+N-1$, and the unidirectional bus throughput is therefore bounded from above by 1.

Next, consider a system (called SYS2) consisting of N stations and two unidirectional buses: one is used to transmit packets from right to left, and the other used to transmit packets in the reverse direction. The activity of a station on one bus is independent of its activity on the other bus. Thus, for example, a station may transmit on one bus while receiving on the other. Now, it is obvious that the capacity of SYS2 is bounded by twice the capacity of SYS1. Also, the capacity of the bidirectional bus system must be bounded from above by the capacity of SYS2 (simply because the stations in SYS2 are less restricted), and thus the claim follows. ■

We therefore conclude that the throughput of any scheduling policy is bounded by:

$$S \le \min(N/2, 2).$$

3.2 Lower Bounds on the System Capacity

In this section, we present lower bounds for the system capacity under various constraints. First we look at an unconstrained system. It is relatively simple to construct a schedule under which the system throughput is $2-2/N$. This schedule is depicted in Figure

2, where a transmission point is represented by a solid dot and a reception point by a hollow square.

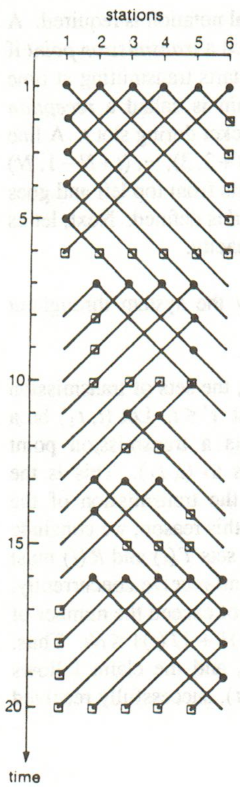

Figure 2: Throughput of Value 10/6 is Attainable on a Six Station System

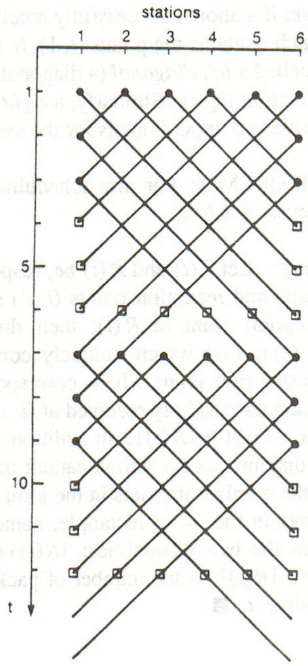

Figure 3: Strictly Fair Throughput of Value 30/20 is Attainable on a Six Station System

For clarity, a packet propagation is represented only by a line (representing the "front" part of the packet), and not by a band. Note that this schedule does not obey the fairness restriction, since most of the traffic is originated from and destined for the end stations (1 and N).

Next we consider fair policies and strictly fair policies. Obviously, the capacity of these systems is bounded by the capacity of the unconstrained system. Surprisingly, we find that even the strictly fair system may achieve a throughput which is very close to 2. In [Levy84], we constructed a strictly fair policy which achieves a throughput of

$$S = 2 - \frac{3}{N+2} - \frac{2}{(N+2)(N-2)}$$

A more efficient transmission pattern for the strictly fair system has been suggested by C. Ferguson [Ferg83]. This pattern is depicted in Figure 3. The throughput attained by this pattern can be calculated by observing that the time to complete the pattern is given by $2+3+\cdots+N = (N+2)(N-1)/2$. The number of packets transmitted in the pattern is $N(N-1)$ (from every station to every other station), and thus the throughput is:

$$S = 2 - \frac{4}{N+2}$$

While fairness does not impose a serious capacity degradation, politeness does. To understand why politeness decreases capacity, one should examine Figures 2 and 3 (where the schedules shown do not obey the politeness restriction). As a matter of fact, it is evident that these schedules "benefit" from letting a station transmit while hearing a packet which is not destined for itself. This degradation is stated in the next theorem.

THEOREM 3: The capacity of a polite system is exactly 1.

Proof: First we show that the system capacity is upper bounded by 1. This can be shown by examining the space-time domain and observing that on any left diagonal there can be at most one transmission point, or else the system does not obey politeness. Therefore, the number of transmission points on the $N \times t$ rectangle is bounded by $T(t) \le t+N-1$. Thus, since the number of packets successfully received by time t is bounded from above by $T(t)$, the system capacity is bounded from above by 1. Now, it is easy to see that a throughput of value 1 is attainable in the system. This can be achieved by having station 1 transmitting all the time, with all the other stations silent. Thus, we conclude that the capacity of the system is exactly 1. ■

Having calculated the capacity of a polite system, we next discuss the capacity of a unidirectional polite system. We claim that if the direction of politeness can be chosen for every station independently of the politeness direction chosen for the other stations, then the capacity of the system can approach 2. To verify this claim, observe Figure 2: Let station 1 be polite to the left and station 6 be polite to the right (which actually implies no politeness for these stations). Let station 2 be polite to the left and station 5 be polite to the right, and let stations 3 and 4 be either polite to the right or polite to the left. Under this politeness rule, the transmission policy depicted in Figure 2 is still valid and the system throughput can get as high as $2 - 2/N$.

On the other hand, if the politeness direction is chosen to be uniform (i.e., either all stations are polite to the left or all stations are polite to the right), then the system capacity remains 1. This claim may easily be proved along the same lines as Theorem 3.

3.3 Discussion

From this analysis, it is evident that the potential of the fast bidirectional bus system is relatively high. The capacity of similar single shared-channel systems, like the one-hop packet radio network or the relatively-slow bidirectional bus system, is known to be 1. In comparison, we showed above that the space-time event separation observed in the very-fast bus system allows the throughput of this system to get as high as 2. This is shown to hold even if (strict) fairness is required in the system.

Surprisingly, however, we realize that forcing the politeness property, which often *increases* the throughput of a bus system under stochastic arrivals (as in the CSMA access scheme), actually *decreases* the system capacity down to 1. Nevertheless, applying directional politeness does not necessarily degrade the system capacity.

4. The System Throughput Under Stochastic Arrivals

The system model is the one given in Section 2, above. The arrival process is modeled according to the "traditional" model of packet radio networks used in the literature (see, for example, [Abra73]). According to this model, the packet transmissions of each station are modeled as a sequence of *independent* Bernoulli trials. This sequence represents the combined stream of old, retransmitted packets and newly arriving packets. Thus we have:

$G_i = Pr$ [ith station transmits a packet in any given slot] ,

$$i = 1,2,\cdots,N$$

Since, in our model, there is importance in the packet destination*, we identify the destination of each packet sent:

$r_{ij} = Pr$ [station i 's packet is destined for station j] , $\quad j \neq i$

This definition obviously requires: $\sum_{j \neq i} r_{ij} = 1$ for $i = 1,2,\cdots,N$.

Two important parameters are considered in this model: the *average traffic* (per slot) (also called the *offered load),* and the *throughput.* The offered load of station i is the expected number of packets (per slot) transmitted by this station. This is denoted above by G_i. Similarly, the offered load from station i to station j, denoted by G_{ij}, is the expected number of packets transmitted from station i to station j. The total offered load of the system, denoted by G, is the expected number of packets transmitted (per slot) in the system. Obviously, we have $G_{ij} = G_i\, r_{ij}$ and $G = \sum_{i=1}^{N} G_i$.

In a similar way, we define the system throughput. The throughput of station i, denoted by S_i, is the expected number of packets (per slot) originated at station i and successfully received at their destination. Similarly, the throughput from station i to station j, denoted by S_{ij}, and the total system throughput, denoted by S, are defined. Note that this definition of throughput is consistent with the definition given in Section 3, above.

4.1 Exact Throughput Analysis of a Non-Polite System

We start the throughput analysis of the system by studying the non-polite scheme. In this scheme, the behavior of one station is independent of the transmissions of the other stations; thus, the throughput from station i to station j can easily be shown to be:

$$S_{ij} = G_i\, r_{ij} \prod_{k \neq i}^{N} (1 - G_k) \qquad i \neq j\,, \tag{3.1}$$

and the total throughput originated at station i is:

$$S_i = \sum_{j \neq i} S_{ij} = G_i \prod_{k \neq i} (1 - G_k) \qquad i = 1,2,\cdots,N\,. \tag{3.2}$$

It is important to emphasize that the model considered here is significantly different from the one considered by [Abra73]. Nevertheless, the basic assumption that the stations' behaviors are independent of each other (by the assumption that no politeness is used) leads both models to the same results. Thus, the throughput of our system is identical to that of the Slotted Aloha system, and we refer the reader to the literature (see, e.g., [Klei76]) for further analysis of its performance.

4.2. Polite System: An Exact Analysis

For the analysis of the polite systems, we must change our assumption on the arrival process. Rather than using the previous Bernoulli assumption, according to which station i is assumed to transmit a packet with probability G_i during *every slot*, we use a modified assumption according to which station i transmits with probability G_i in each slot in which it is not forced to be silent by the politeness rule. Thus, if we observe the slots in which station i is allowed to transmit by the politeness rule, the packets transmitted from station i behave like a stream of Bernoulli trials.

Under these assumptions, it is possible to represent the system behavior by a Markov chain. However, note that the stations' status is not sufficient to represent the system. Rather, in order to form a Markov chain, we need to include the status of the channel during the tth slot in this representation. A state in this Markov chain can be described by the channel status at each of its $N-1$ segments, where a segment is the channel section between two neighboring stations. During slot t, each of these segments may be in one of four states: a) no transmission propagates on the segment, b) transmission from left to right propagates along the segment, c) transmission from right to left propagates along the segment, and d) two concurrent transmissions (from left and from right) propagate along the segment. Since the number of segments is $N-1$, the state space contains 4^{N-1} states.

For very small values of N ($N \leq 5$), it is possible to solve this Markov chain by calculating the (finite) transition matrix and numerically solving for the steady state probabilities of the system states. To demonstrate the method, consider a two station system. During each slot, the channel may be in any one of four states: a) only station 1 transmits, 2) only station 2 transmits, 3) both stations transmit and 4) neither of the stations transmit. We denote the probability that the system is in these states during slot t by $\pi_1(t)$, $\pi_2(t)$, $\pi_{12}(t)$, and $\pi_0(t)$, respectively. These probabilities obey:

$$\pi_0(t+1) = (1-G_1)(1-G_2)\pi_0(t) + (1-G_1)\pi_1(t) + (1-G_2)\pi_2(t) + 1 \times \pi_{12}(t)$$

$$\pi_1(t+1) = G_1(1-G_2)\pi_0(t) + G_1\pi_1(t)$$

$$\pi_2(t+1) = (1-G_1)G_2\pi_0(t) + G_2\pi_2(t)$$

$$\pi_{12}(t+1) = G_1 G_2 \pi_0(t)$$

Under steady state, we may drop the time reference from these equations and solve the resulting equation set for the steady state probabilities (denoted by π_1, π_2, π_{12} and π_0) in terms of the system parameters (G_1, G_2). From this solution, we then get the system throughputs: $S_{12} = \pi_1 + \pi_{12}$, $S_{21} = \pi_2 + \pi_{12}$. Note that the throughput must obey $S_{12} + S_{21} \leq 1$, due to Theorem 3.

To demonstrate the system behavior, we next analyze a three station system. Two symmetry assumptions are used in this analysis: 1) The transmission rate G_i for two symmetrically positioned stations is assumed to be identical, and thus we assume that

*The destination information is not important in the traditional model of a slow bus network, since the successful reception of a packet does not depend on its destination.

$G_1 = G_3 = p$ and $G_2 = q$; 2) The destination of a packet transmitted from station i is equally likely to be any of the other $N-1$ stations, i.e., $r_{ij} = 1/(N-1)$ for $j \neq i$ and $r_{ii} = 0$.

Using the method described above, we construct the Markov chain (consisting of sixteen states) representing the system (see [Levy84]), solve it numerically and calculate the system throughputs. The results of this analysis are depicted in Figures 4 and 5. Figure 4 is a three dimensional plot of the throughput originated at a side node and destined for the other side node (the sum of S_{13} and S_{31}) as a function of p and q. Figure 5 depicts the total throughput (S) in the system as a function of p and q. A discussion of the system behavior as observed in these figures is given in Section 4.4.

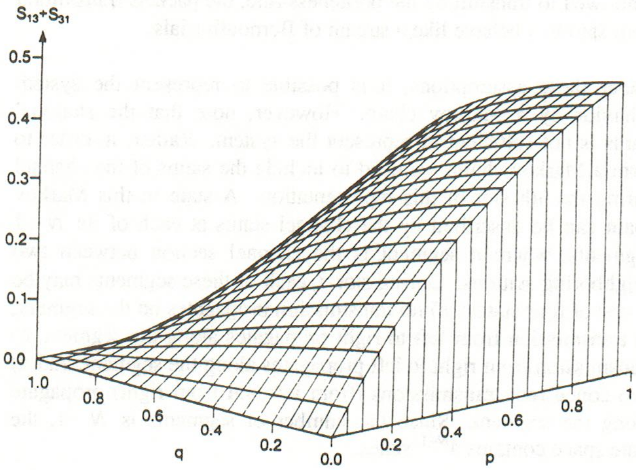

Figure 4: The Side-to-side Throughput in a Three Station System

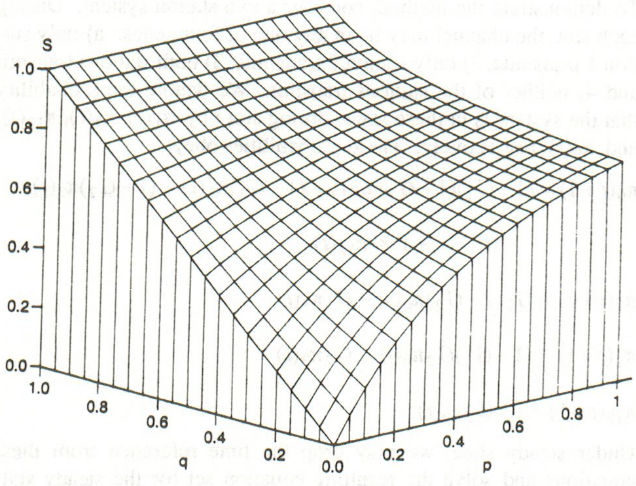

Figure 5: The Total Throughput in a Three Station System

4.3 Polite System: An Approximation for an N Station System

Since the exact method described in the previous section may not be applied for systems with large N (due to the exponential number of equations -- 4^{N-1}), we next propose an alternative approximation method.

Let the triple (RS,k,t) (the triple (LS,k,t)) denote the event that during slot t, station k hears a packet arriving from the right (left). Let the triple (Q,k,t) denote the event that station k is quiet (does not transmit) during slot t. To derive the system throughput, we first calculate the probability that the event (LS,k,t) occurs. This event occurs *if and only if* for every station j, such that $1 \le j < k$, station j does not transmit at time $t+j-k$. Thus

$$Pr[(LS,k,t)] = Pr[(Q,k-1,t-1), (Q,k-2,t-2),\cdots,(Q,1,t-k+1)] \tag{4.1}$$

This can be calculated as:

$$Pr[(LS,k,t)] = Pr[(Q,k-1,t-1) \mid (Q,k-2,t-2),\cdots,(Q,1,t-k+1)] \times Pr[(Q,k-2,t-2),\cdots,(Q,1,t-k+1)] \tag{4.2}$$

The conditional probability given above can be calculated as follows:

$$Pr[(Q,k-1,t-1) \mid (Q,k-2,t-2),\cdots,(Q,1,t-k+1)] = 1 - G_{k-1} \times Pr[(RS,k-1,t-1) \mid (Q,k-2,t-2),\cdots,(Q,1,t-k+1)] \tag{4.3}$$

Now, to calculate the expression

$$Pr[(RS,k-1,t-1) \mid (Q,k-2,t-2),\cdots,(Q,1,t-k+1)]$$

we make the following independence assumption.

INDEPENDENCE ASSUMPTION: The event (RS,k,t) is independent of the events $(Q,k-1,t-1),\cdots,(Q,1,t-k+1)$.

This assumption means that the event: {station k hears a transmission arriving from the right at time t}, is independent of the event: {stations $k-1, k-2, \cdots, 1$ are quiet at times $t-1, t-2, \cdots, t-k+1$, respectively}. Obviously, this is not a true property of our system, since these events are correlated to each other. However, it is easy to see that the dependency between these events is relatively weak, and thus we assume full independence.

We now assume that the system is at steady state, and denote $R_k \triangleq Pr[(RS,k,t)]$, $L_k \triangleq Pr[(LS,k,t)]$. Then, from the independence assumption and from Equations (4.2) and (4.3), we may conclude:

$$L_k = (1-R_{k-1}\,G_{k-1})\,(1-R_{k-2}\,G_{k-2})\cdots(1-R_1\,G_1) \;; \quad k = 1,2,\cdots,N-1 \tag{4.4}$$

In a symmetric way, we calculate R_k:

$$R_k = (1-L_{k+1}\,G_{k+1})\,(1-L_{k+2}\,G_{k+2})\cdots(1-L_N\,G_N) \;; \quad k = 2,3,\cdots,N \tag{4.5}$$

The values of R_1 and L_N are obviously 1. Now Equations (4.4) and (4.5) form a set of $2N-2$ equations in $2N-2$ variables, a set which can be solved by numerical methods.

Using the independence assumption and assuming steady state (see [Levy84]), we may now calculate the system throughput as a function of the parameters R_k and L_k:

$$S_{jk} = G_j\, r_{jk}\, L_j\, R_j\, R_k \;; \quad j < k \tag{4.6a}$$

$$S_{jk} = G_j\, r_{jk}\, R_j\, L_j\, L_k \;; \quad j > k \tag{4.6b}$$

40.5.6.

From these equations and from (4.4) and (4.5), one can calculate the total system throughput as a function of the transmission parameters.

Next we examine the quality of the approximation. We do so by computing the throughput for systems where the offered load is identical for all stations ($G_i = p$). For the three station system, the approximation results are compared to the exact results (derived in Section 4.2) and depicted in Figure 6. For a ten station system, the approximation results are compared to simulation. Figure 7 depicts the throughput in a ten station system; shaded dots represent simulation, and each curve represents the sum of the throughput for two symmetric stations (e.g., 1 and 10).

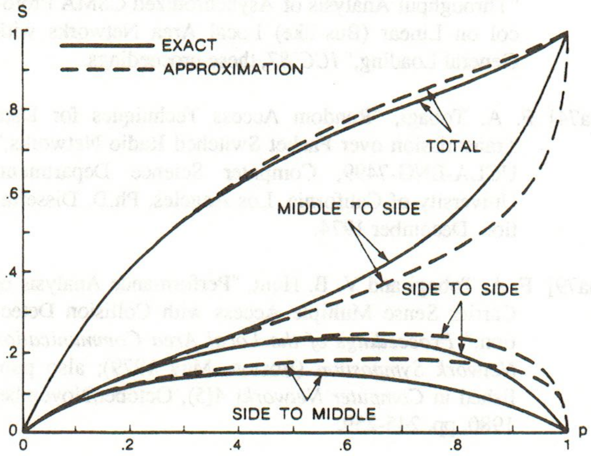

Figure 6: The Throughput in a Fully Symmetric Three Station System

From these comparisons, we observe that the approximation predicts the individual station throughput quite accurately for low offered loads ($p < 0.6$) and not so accurately for higher offered loads. The reason is that at high loads, the dependency between events increases and thus the independence assumption does not reflect the system behavior properly. Note, however, that the accuracy increases with the system size, so that for large systems the approximation may be quite accurate. In light of the discrepancies in predicting the individual station throughput, the predictions for the total system throughput are, surprisingly, very accurate. It seems that the errors in predicting the individual station throughput, using the independence assumption, compensate for each other, yielding a very good approximation for the total throughput.

4.4. Discussion of the Results

The analysis of this section reveals the important properties of the very-fast bus system. These properties are discussed below.

From the analysis of the three station system, we see how the system throughput is affected by the offered load of the individual stations. At the level of individual stations we recognize that when a given station increases its load, the throughput originated at this station will increase, while the throughput originated at the other stations will decrease. This behavior is quite common for shared channel communication networks; for example, the slotted ALOHA system and the non-polite system described in Section 4.1, above, behave the same way (see Equation (3.2)).

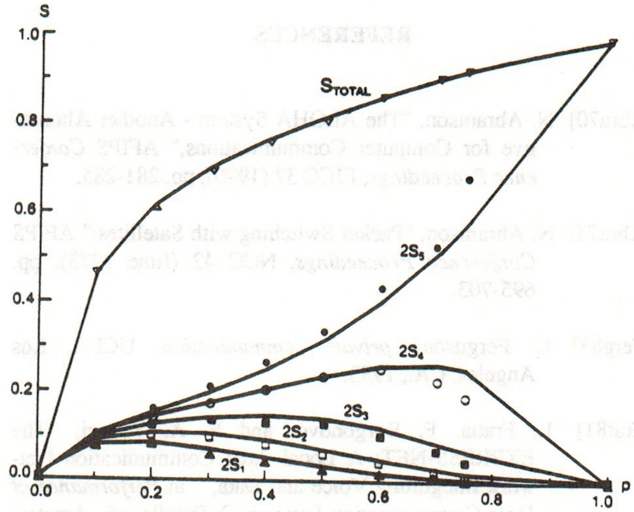

Figure 7: The Throughput in a Fully Symmetric Ten Station System: Simulation vs. Approximation

While at the individual station level, the polite system behaves very much like other shared channel systems; the advantages of this system are revealed by examining the behavior of its total throughput. From Figure 5 we observe that any increase in the offered load, either of the side stations or of the middle station, causes an increment in the total throughput. The importance of this property is that the system is very stable; whenever the system load increases, the throughput also increases. This property is not very common in shared channel communication networks. For example, in slotted ALOHA, which is unstable (see, e.g., [Klei76]), an increase in the offered load may cause the total throughput to decrease.

The importance of the stability property is that no special mechanisms are required for controlling the system stability. In non-stable systems, such as slotted ALOHA, one must control the offered load to prevent the system from getting into unstable situations (situations in which the system blocks itself); here these mechanisms are not required, since the system controls itself in a natural way.

The explanation for this stability property can be given by observing that, unlike other shared channel systems, the stations in this system are not all alike. Rather, at every moment, t, some stations get transmission priority over others. More specifically, we may note that if station i successfully transmitted at time $t-1$, it has full transmission priority at time t (due to the politeness), and thus, if it does transmit at this time, the transmission will be successful as well. This type of behavior leads the system to behave in an "exhaustive" fashion, in which a station that grabs the channel will hold it for quite a while, while the other stations remain polite.

When the system is fully symmetric, its behavior is very similar. Figures 6 and 7 show that at low load the throughput of every station increases, while at high load the middle stations become more and more dominant because the side stations become more quiet. The total throughput, nonetheless, monotonically increases with the offered load.

While these properties have been observed with regard to the several systems we studied, it remains as an open question whether or not the properties hold for any size system.

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