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Poisson winner queues

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Abstract

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We study special types of queues, winner queues, in which *all customers are served concurrently*. A customer in a winner queue will successfully finish his service (i.e. "win") and leave if no other customer leaves during his current service. Once a customer wins, all others in service at that time "lose"; thus we have a situation in which concurrent customers conflict, yielding a single "winner". There are four disciplines considered: silent-redraw, silent-noredraw, broadcast-redraw, and broadcast-noredraw.

The winner queues considered have an infinite number of servers. We assume that service times consist of a deterministic part and an exponential part. This type of service time distribution includes pure deterministic and pure exponential service times as special cases.

Using a one-dimensional imbedded Markov chain and a recursive formula for the state probabilities, we obtain numerical results for certain disciplines and distributions of requested and restarted service time. For some broadcast winner queues we show analytic results. In all cases we also give simulation results which indicate the correctness and accuracy of our numerical calculations.

The results obtained are given in terms of the normalized average system time and the normalized power (defined as the system load divided by the normalized average system time). One application of this model is to study the performance of optimistic concurrency control schemes in databases.

Keywords: multiuser queues, numerical procedures, optimistic concurrency control, queueing, special queues, transition probabilities, one-dimensional Markov chain.

1. Introduction

The goal of this article is to analyze special types of queues, which we call "winner queues". Winner queues are designed to mimic the behavior of transactions in a database system. Here we consider simpler forms of these queues; more general winner queues are analyzed in [4]. Numerical results found for winner queues are used in [4] for performance evaluation of optimistic concurrency control (OCC) in databases. (For a discussion of OCC schemes, see, for example, [3,8].) Tsitsiklis, in [7], has looked at another type of queue which may be used for the analysis of concurrency control based on static locking, a nonoptimistic scheme.

In the next section we define the model of winner queues. We describe and discuss simulation results in Section 3, and develop an analytical approach for winner queues in Section 4. Numerical results for different winner queues are given in Section 5.

2. Model

Consider a regular G/G queue with an arbitrary interarrival time density $a(t)$, and an arbitrary service time density $b(x)$. Assume now that each of the customers accesses the same resource. Let a given customer i start his service at time t_i (i.e., at his arrival time to the system); with service time X. If during the time interval $(t_i, t_i + X)$ no other customers leave the system, then, at time $t_i + X$, customer i will finish his service successfully and leave the system. In this case we say that customer i *wins*. If, on the other hand, some other customer j wins and leaves the system in $(t_i, t_i + X)$, say at time $t_j + X_W$, where $t_i < t_j + X_W < t_i + X$, then customer *loses*. In the former case, customer i is called a *winner*, while in the latter case, customer i is called a *loser* and customer j is a winner. Every time a customer loses, he restarts his service, and he does that over and over again until he finally wins. A queue with this discipline we call a *winner queue*.

We differentiate between two types of queues, depending on the behavior of the system upon the departure of a winner. Consider again that customer i starts his service at time t_i, with service time X, and that customer j wins at time $t_j + X_W$, $t_i < t_j + X_W < t_i + X$. If, at time $t_j + X_W$, the system notifies all the other customers about the departure of winning customer j, then, customer i immediately learns that he lost, and restarts his service at once, i.e., at time $t_j + X_W$. In this case we say that the system *broadcasts* that a departure took place. We call this queue a *broadcast winner queue*. If the system does nothing upon a departure of a customer, i.e., if it remains *silent*, then all other customers do not learn that they lost until they finish their present service. This means that our loser i from the above example will restart not at time $t_j + X_W$, but at time $t_i + X$. A winner queue with this behavior we call a *silent winner queue*.

In addition to winner queues being silent or broadcast, the properties of service times *upon restart* of losers divide all winner queues into *redraw* and *noredraw* winner queues. In the redraw queues the service time of each restart is redrawn independently from the same service time density $b(x)$. Our loser, customer i, from the above example, will, then, be scheduled to finish his restarted service at time $t_j + X_W + X_r$, in the broadcast case, or at time $t_i + X + X_r$, in the silent case, where X and X_r are drawn from the density $b(x)$. In the noredraw queues, service times upon each restart are equal to the initial service time (they are *not redrawn*) [1]. So, the loser i, in such a system, will be rescheduled to finish his restart at time $t_j + X_W + X$, in the broadcast case, or at time $t_i + 2X$, in the silent case.

The winner queues described above have identical requested and restarted service time distributions. They are a special case of general winner queues considered in [4] in which the mean and distribution of the restarted service times, in general, *differ* from the requested service time distribution.

[1] In general, we may allow restarted service times to be only a fraction of the initial service time, but in this article we make the restriction of the fraction being equal 1.

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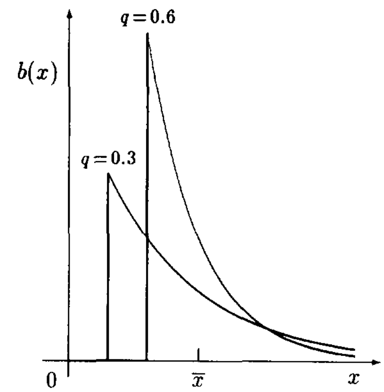

Fig. 1. D_qM probability density.

We define service time requirements in the winner queues as the sum of a deterministic and an exponential part. The distribution of such random variables we call D_qM, where q represents the fraction of the mean service time which is deterministic. The probability density of such service times is shown in Fig. 1. Its analytical form is:

$$b(x)=\begin{cases}0, & 0 \leqslant x \leqslant q\bar{x}\\ \frac{\mu}{p}\,\mathrm{e}^{-(\mu x-q)/p}, & x>q\bar{x},\end{cases} \tag{1}$$

where $p=1-q$ and $\mu=1/\bar{x}$. Pure exponential and pure deterministic distributions are special cases of the D_qM distribution (for $q=0$ and $q=1$, respectively). Note that D_qM is a delayed exponential distribution. The distribution above was used, under a different name, by Ševcik in [6], in the context of concurrency control techniques.

The system load ρ we define as the ratio of the average service time $\bar{x}$ and the average interarrival time $\bar{t}$, $\rho=\bar{x}/\bar{t}=\lambda/\mu$, where λ is the average arrival rate, and μ the average service rate. Let T represent the average system time (the time an average customer spends in the system). We define the normalized average system time as $T_n=T/\bar{x}$, and the normalized power $P=\rho/T_n$, see [2]. Clearly, we wish to maximize power. While there may be interest in other forms of power functions, such as $P=\rho^k/T_n$, for $k\neq 1$, we choose to look only at a simple one, for $k=1$, as another representation of performance of the winner queues, in addition to T_n. [2] Note that for any ρ, the smallest normalized average system time is 1 (no waiting); thus an upper bound on P is $P \leqslant \rho$, which corresponds to a *perfect* system. An example of a perfect system is D/D/1, when $\rho<1$.

We are interested in evaluating the performance of winner queues with Poisson arrivals and the following four disciplines: silent-redraw (denoted as SR), silent-noredraw (SN), broadcast-redraw (BR), and broadcast-noredraw (BN). The performance measures we consider are the normalized system time and the normalized power, as well as the distribution of the number of customers in the system.

3. Simulation results

Simulation results for the four types of winner queues have been obtained. For each of the queues, q was varied from 0 to 1.

Figures 2 and 4 show [2] that redraw queues with more deterministic service times, i.e., with higher q, perform worse than redraw queue with lower q. Redraw queues with pure exponential service times give

[2] We specify winner queues in the same form as any other queues, with the addition of discipline code, for example (SR). All winner queues considered have an infinite number of servers, and so we omit the ordinary specification of the number of servers. Note that for queues whose service time distribution is deterministic, redraw and noredraw cases are equivalent. Such is the queue M/D(S).

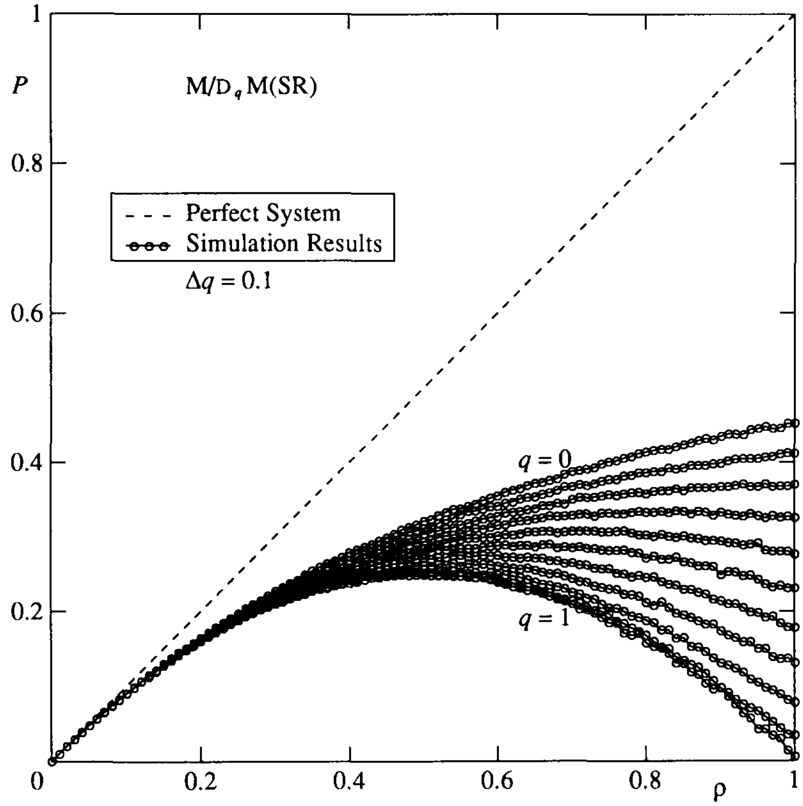

Fig. 2. Simulation results for the power (P) in M/D$_q$(M(SR).

the highest power, while pure deterministic give the lowest power. Quite the opposite is the situation with noredraw queues, as illustrated in Figs. 3 and 5. Here, the worst performance is for queues with $q = 0$ (pure exponential). The initial service times are independent of whether the queue is redraw or noredraw. It is the service times upon restarts that affects the performance differently. In redraw queues, service times upon restart tend to be smaller for smaller q due to the nature of service time probability distribution. In noredraw queues, however, customers with long initial service times will negatively affect the average response time because they have a poor chance of winning and their service times upon restart will stay fixed at the initial high value. Furthermore, the smaller is q, the higher is the probability of long service time. Thus, noredraw queues with less deterministic service times perform worse than noredraw queues with more deterministic service times.

From Figs. 2 through 5 we see that redraw queues perform better than noredraw queues. Again, this is due to the nature of the service times. Figure 2 shows that redraw of service times will cause probabilistic shortening of service times and, thus, will result in better performance of redraw queues.

Since in broadcast queues unsuccessful services are terminated even before their prescheduled completion, these systems perform better than silent systems. Broadcast systems have superior performance compared to silent systems.

It is interesting to note that for the M/M(SR) system(s) the normalized power does not drop with an increase in ρ; this is somewhat visible in Fig. 2, but can be seen more clearly in Fig. 11 below. In fact it seems that the power approaches a constant as ρ goes to infinity. Such behavior of the system is due to the redraw of service requests upon restart and to the memoryless nature of the service time distribution. Successful service times are shorter than the requested service times, and for high ρ they tend to zero. In all cases, the mean queue length settled down for finite ρ. From Fig. 11 below we will find that the average system time seems to grow linearly with ρ.

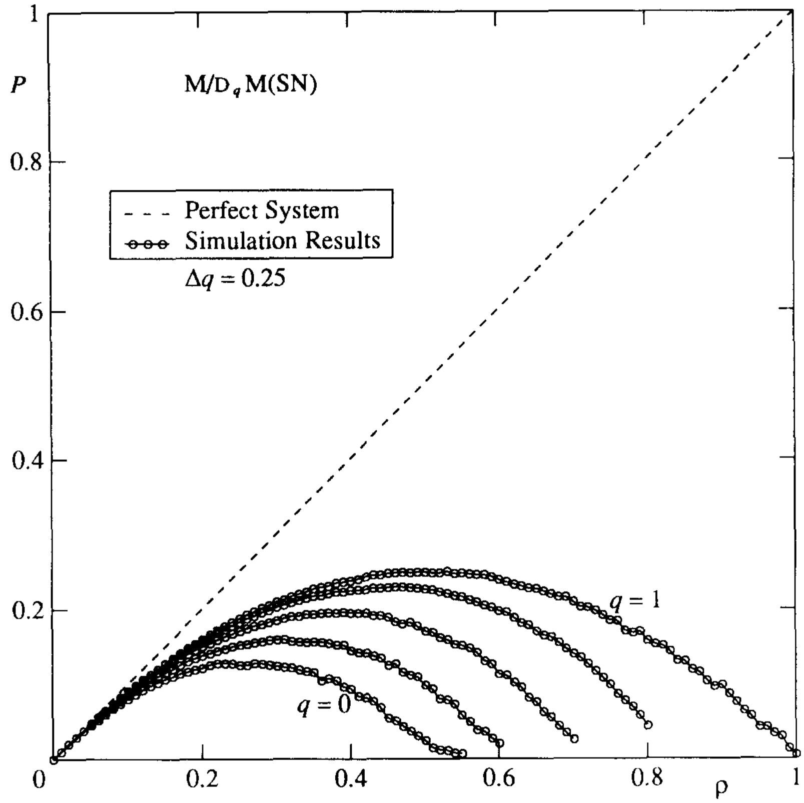

Fig. 3. Simulation results for the power in $M/D_qM(SN)$.

From Fig. 4 we see that the M/M(BR) queue gives performance values close to perfect. (M/M(BR) is shown as the $M/D_qM(BR)$ curve for $q = 0$.) In Section 5.3 we will see that M/M(BR) indeed gives perfect performance.

Now that we have seen the simulation results (shown in Figs. 2 through 5) and understand the differences in the behavior of the four types of Poisson winner queues (with a D_qM service time distribution), we proceed to analyze some of these systems below.

4. Analysis

In order to find the normalized power in Poisson winner queues, we use an imbedded Markov chain to calculate the distribution of the number in system left by departing customers. We define $D(n)$ to be the number of customers left behind by the nth departing customer. $D(n)$ is an imbedded Markov chain whose distribution we seek. We define d_k to be the equilibrium probability that k customers are left in the system by a departure, i.e., $d_k \overset{\text{def}}{=} \lim_{n\to\infty} P[D(n) = k]$. Since arrivals are Poisson and since we have unit changes in states, then d_k also gives the equilibrium distribution for the number in system at all times. Thus, once we have calculated all the d_k, $k = 1, 2, \ldots,$ we can find the average number in system, N, that is, $N = \Sigma k d_k$.

Using Little's result, we have the normalized average system time

$$T_n = \frac{N}{\rho}, \tag{2}$$

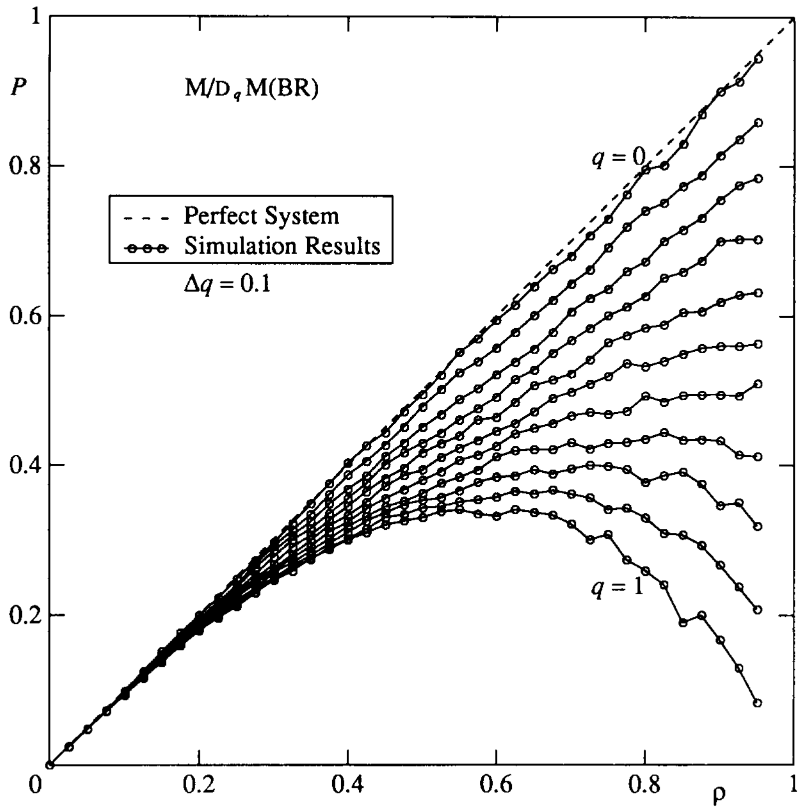

Fig. 4. Simulation results for the power in $M/D_qM(BR)$.

where $\rho = \lambda \bar{x}$. The normalized power P is calculated as

$$P = \frac{\rho}{T_n} = \frac{\rho^2}{N}. \tag{3}$$

Having defined the states of the imbedded Markov chain to be the number of customers in the system left by departures, we further define the transition probabilities between the states, $p_{i,j}$, as follows

$p_{i,j} =$ P[a departure leaves j customers in the system, given the previous departure left i customers in the system], $i, j = 0, 1, 2, \ldots$

From the transition probabilities we can calculate the distribution of the number of customers in the system left by departures from the following equation.

$$d_k = \sum_{i=0}^{\infty} d_i p_{i,k}, \quad k = 0, 1, 2, \ldots \tag{4}$$

In the following section we show that $p_{i,k} = 0$, $i > k + 1$; then, we get the recursive formula

$$d_k = \frac{1}{p_{k,k-1}} \left[d_{k-1} - \sum_{i=0}^{k-1} d_i p_{i,k-1} \right], \quad k = 1, 2, \ldots \tag{5}$$

Below we find expressions for $p_{i,j}$ but they are not in a closed form and so we resort to numerical evaluation procedures.

The numerical procedure for finding the normalized response time and power is as follows. We first truncate the infinite chain d_k by calculating only first M probabilities ($0 \leqslant k \leqslant M$), where M is introduced as an arbitrary positive integer. We assume that all other probabilities (for $k > M$) are equal to 0. By using higher M, we will obtain more accurate results. In fact, the results are asymptotically exact as

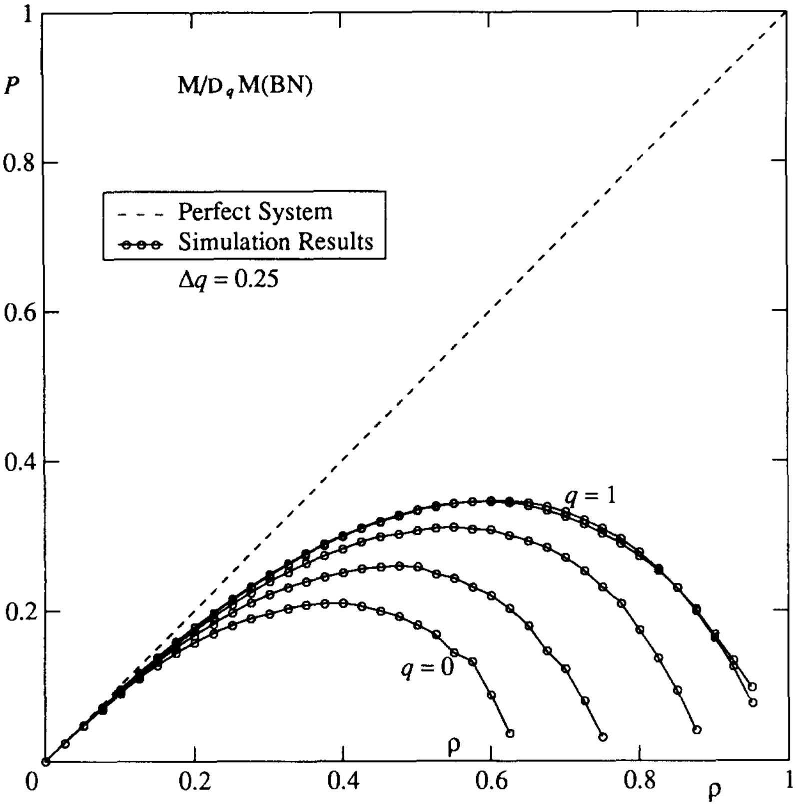

Fig. 5. Simulation results for the power in M/D$_q$M(BN).

M approaches infinity, given the transition probabilities are exact. Thus, we will also refer to the number M below as the *precision* of the numerical solution. In order to obtain d_k, $0 \leqslant k \leqslant M$, we calculate all the transition probabilities $p_{i,j}$, $0 \leqslant i \leqslant M$, $0 \leqslant j \leqslant M-2$. To preserve the conservation of the probability, we assign the following value to the $p_{i,M-1}$, $0 \leqslant i \leqslant M$.

$$p_{i,M-1} = 1 - \sum_{j=0}^{M-2} p_{i,j}, \quad 0 \leqslant i \leqslant M. \tag{6}$$

We assign the value 1 to the probability d_0, and from the recursive formula (5) we find all the probabilities d_k, $1 \leqslant k \leqslant M$. Let the sum of all the d_k, $0 \leqslant k \leqslant M$ be C. We then divide every d_k, $0 \leqslant k \leqslant M$ by C. From the d_ks we find the number of customers in the system N. Through equations (2) and (3) we use N to obtain the normalized average response time T_n and normalized power P.

In the sections below we find the transition probabilities.

4.1. Finding the transition probabilities

The imbedded Markov chain, $D(n)$, with arcs representing transition probabilities, is shown in Fig. 6. Since at most one customer may leave between two successive departures, we have

$$p_{i,j} = 0, \quad j < i-1 \tag{7}$$

and thus those transitions are not shown in Fig. 6

We define the following joint probability for i, $j = 0, 1, 2, \ldots,$ $\quad v > 0$

$p_{i,j}(v)\,\mathrm{d}v =$ P[a departure leaves j customers in the system (given the previous departure left i customers in the system) and the interdeparture time, V, between these two departures lies in the interval $v < V < v + \mathrm{d}v$].

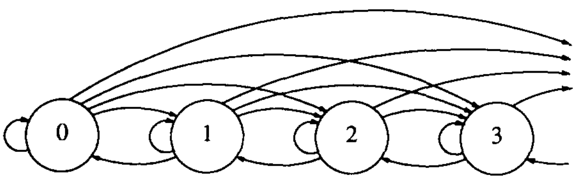

Fig. 6. Poisson winner queue state transition diagram.

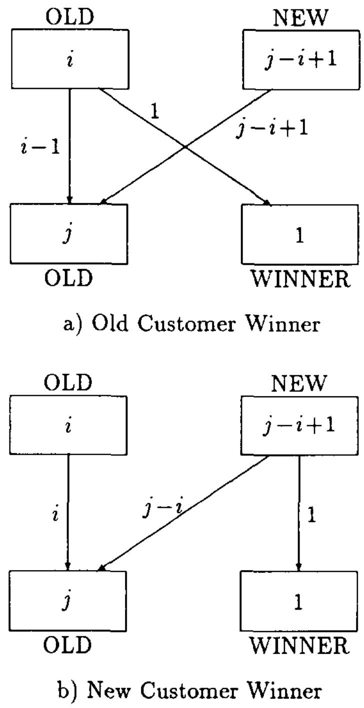

Fig. 7. Poisson winner queue transition graphs.

Once we find the probability $p_{i,j}(v)$, we may find $p_{i,j}$ as

$$p_{i,j} = \int_0^\infty p_{i,j}(v)\,\mathrm{d}v. \tag{8}$$

Consider a departure of a customer from the system. Relative to that departure, we call all the customers left in the system at the departure *old customers*, and any customers that arrive after the departure *new customers*. The first following departure can be made by either an old customer or by a new customer. Figure 7 shows transition graphs for the two cases.

Let us explain the way the transition graph (a) in Fig. 7 is constructed. The graph represents a transition from state i to state j. We draw two rows of boxes. The first row is associated with the state i. One of the boxes represents old customers, and we write "i" in it. The other box represents new customers that arrived before the transition to the state j. We leave that box empty for now. The second row of boxes is associated with the state j. In the box that represents old customers we write "j", and in the box that represents a winner we write "1". We now draw arcs from the boxes in the first row to the boxes in the second row. The labels on the arcs represent the number of customers that are transferred from one box to another. Since we know that the winner is an old customer (for the case (a)) we draw an arc labeled "1" from the top old customer box to the winner box. We know that all the other old customers remained old, and so we draw an arc labeled "$i-1$" from the top old customer box to the bottom old customer box. Next we know that all other $j-i+1$ old customers at state j must have newly arrived, and so we draw and arc labeled "$j-i+1$" from the new customer box to the bottom old customer box. We have now completed drawing arcs since the sum of the labels on the arcs equals the sum of the bottom row of boxes. Now we take the sum of the labels of all the arcs that leave the new customer box, and we write that number in the box, i.e., we write "$j-i+1$". In a similar way we draw the transition graph for the case (b) in Fig. 7.

Let us consider again a departure of a customer from the system. Let that customer leave i customers in the system and let that departure occur at time $t = 0$. We define [3] the following four probabilities, P_{OL}, P_{OW}, P_{NL}, and P_{NW}:

$P_{OL}(i, v)$ = P[none of the i old customers finish their service in the interval $(0, v)$ (given i old customers in the system at $t = 0$)],

$P_{OW}(i, v)\,\mathrm{d}v$ = P[$i-1$ out of the i old customers finish their next service after time v, and one old customer finishes his next service in the interval $(v, v + \mathrm{d}v)$ (given i old customers in the system at $t = 0$)],

$P_{NL}(k, v)$ = P[k new customers arrive in $(0, v)$ and none of them finish their service before time v],

$P_{NW}(k, v)\,\mathrm{d}v$= P[$k$ new customers arrive in $(0, v)$, $k-1$ of them finish service after time v, and one of them finishes in the interval $(v, v + \mathrm{d}v)$].

The interdeparture time probability density, given all new customers lost, is simply $P_{OW}(i, v)$. Multiplying that by the probability that all new customers lost, gives us the interdeparture time probability density for an old customer winner: $P_{OW}(i, v)P_{NL}(j-i+1, v)$ as can be clearly observed from part (a) of Fig. 7. From part (b) of the same figure, we see that the interdeparture time density, given all old customers lost, is $P_{NW}(j-i+1, v)$. Unconditioning with $P_{OL}(i, v)$, we obtain the interdeparture time density for a new customer winner: $P_{OL}(i, v)P_{NW}(j-i+1, v)$. We can now write

$$p_{i,j}(v) = P_{OW}(i, v)P_{NL}(j-i+1, v) + P_{OL}(i, v)P_{NW}(j-i+1, v), \quad i, j \geqslant 0. \tag{9}$$

4.2. Finding $P_{OL}(i, v)$ and $P_{OW}(i, v)$

Consider an old customer left in the system by a departure at time zero. We define U_{old} to be a random variable representing the time until the end of his present (unsuccessful) service, and $V_{old} = U_{old} + X_r$ to be a random variable representing the time until the end of his restarted service. X_r is the restarted service time. We here assume the redraw case and so X (the service time) and X_r are both drawn independently from $b(x)$. (This approach would also be applicable for the noredraw case if we knew the distribution of V_{old} for the noredraw systems.) Let $\mathscr{U}_{old}(v)$ represent the probability distribution function of the random variable U_{old}. Let $\mathscr{V}_{old}(v)$ represent probability distribution function of the random variable V_{old}. The following holds [4]:

$$\begin{aligned} P[V_{old} \leqslant v] &\overset{\text{def}}{=} \mathscr{V}_{old}(v) \\ &= \mathscr{U}_{old}(v) \otimes b(v) \\ &= \int_0^v \mathscr{U}_{old}(u) b(v-u)\,\mathrm{d}u, \end{aligned} \tag{10}$$

$$P_{OL}(i, v) = (P[V_{old} > v])^i = [1 - \mathscr{V}_{old}(v)]^i, \tag{11}$$

$$P_{OW}(i, v) = i\frac{\mathrm{d}P[V_{old} \leqslant v]}{\mathrm{d}v}(P[V_{old} > v])^{i-1} = -\frac{\mathrm{d}}{\mathrm{d}v}P_{OL}(i, v). \tag{12}$$

The service time probability density function $b(x)$ is assumed to be known. Let $B(x)$ be the corresponding probability distribution function. We now only need to find $\mathscr{U}_{old}(u)$ in order to find $P_{OL}(i, v)$.

For broadcast systems $U_{old} = 0$, and thus we have

$$\mathscr{U}_{old}(u) = 1, \quad u \geqslant 0,$$

which gives us, in the broadcast case

$$P_{OL}(i, v) = (P[X > v])^i = [1 - B(v)]^i. \tag{13}$$

[3] The letters O, N, W, and L stand for "old", "new", "win", and "lose", respectively.

[4] $\otimes$ represents convolution.

We now assume that the service time for the redraw systems has a D_qM distribution as defined in equation (1); we then have:

$$P_{OL}(i, v) = \begin{cases} 1, & v \leqslant q\bar{x} \\ e^{-i(\mu v - q)/p}, & v > q\bar{x}, \end{cases} \tag{14}$$

where μ is defined as $\mu \stackrel{\text{def}}{=} 1/\bar{x}$ and $p = 1 - q$. Using equation (12) we get

$$P_{OW}(i, v) = \begin{cases} 0, & v \leqslant q\bar{x} \\ i\dfrac{\mu}{p}\, e^{-i(\mu v - q)/p}, & v > q\bar{x}. \end{cases} \tag{15}$$

For silent redraw systems with memoryless service times we have

$$\mathscr{U}_{\text{old}}(u) = 1 - e^{-\mu u} = B(u). \tag{16}$$

Using equations (1) and (10) we get

$$\mathscr{V}_{\text{old}}(v) = \int_0^v \mu\, e^{-\mu u}[1 - e^{-\mu(v-u)}]\, du = 1 - (1 + \mu v)\, e^{-\mu v}.$$

Thus, using equation (11)

$$P_{OL}(i, v) = (1 + \mu v)^i\, e^{-i\mu v}, \quad v \geqslant 0 \tag{17}$$

and from equation (12)

$$P_{OW}(i, v) = i\mu^2 v(1 + \mu v)^{i-1}\, e^{-i\mu v}, \quad v \geqslant 0. \tag{18}$$

For the silent winner queues with deterministic service times (equal to $\bar{x}$) we make an approximation by assuming that the arrivals of old customers are memoryless within the time interval $[0, \bar{x}]$, i.e., they are exponentially distributed but also forced to arrive in $[0, \bar{x}]$. This gives us the following approximate expression for $\mathscr{U}_{\text{old}}(u)$.

$$\mathscr{U}_{\text{old}}(u) \cong \begin{cases} \dfrac{1 - e^{-\mu u}}{1 - 1/e}, & 0 \leqslant u \leqslant \bar{x} \\ 1, & u > \bar{x}. \end{cases} \tag{19}$$

For deterministic service times we have

$$B(x) = \begin{cases} 0, & x \leqslant \bar{x} \\ 1, & x > \bar{x} \end{cases}$$

and thus, from equations (10) and (11) we have

$$P_{OL}(i, v) = \begin{cases} 1, & 0 \leqslant v \leqslant \bar{x} \quad \text{or} \quad i = 0 \\ \left(\dfrac{e^{1-\mu v} - 1/e}{1 - 1/e}\right)^i, & \bar{x} < v \leqslant 2\bar{x},\ i \geqslant 1 \\ 0, & v > 2\bar{x},\ i \geqslant 1. \end{cases} \tag{20}$$

From the last equation and equation (12) and we get

$$P_{OW}(i, v) = \begin{cases} \dfrac{i\mu\, e^{1-\mu v}}{1 - 1/e}\left(\dfrac{e^{1-\mu v} - 1/e}{1 - 1/e}\right)^{i-1}, & \bar{x} < v \leqslant 2\bar{x} \\ 0, & \text{otherwise}. \end{cases} \tag{21}$$

In this section we found the exact values for the probabilities $P_{OL}(i, v)$ and $P_{OW}(i, v)$ for $M/D_qM(BR)$ and M/M(SR), and an approximation for M/D(S) [5]. We point out that for $M/D_qM(SR)$ with $q > 0$, we

[5] See footnote 2.

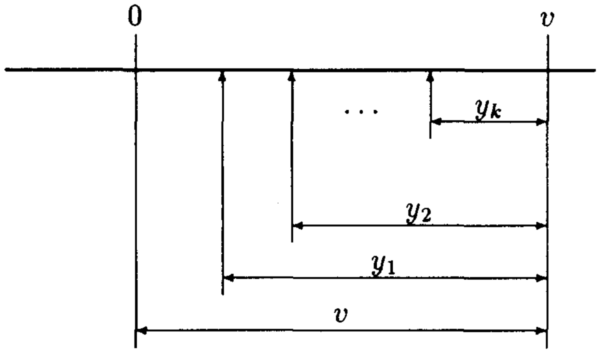

Fig. 8. New arrivals.

have not yet found the probability distribution of the random variable U_{old}, and so we are unable to find $P_{\text{OL}}(i, v)$. For noredraw, except for the approximation M/D(S), we do not have the distribution of U_{old}.

4.3. Finding $P_{\text{NL}}(k, v)$

Figure 8 shows the time axis with k new customers arriving in the interval $(0, v)$. Interarrival times of the customers are: $v - y_1$, $y_1 - y_2, \ldots, y_{k-1} - y_k$. From the definition of $P_{\text{NL}}(k, v)$ and defining $\beta(x) \overset{\text{def}}{=} P[X > x] = 1 - B(x)$, we may write

$$P_{\text{NL}}(k, v) = \int_0^v \lambda\, \mathrm{e}^{-\lambda(v-y_1)} \beta(y_1) \int_0^{y_1} \lambda\, \mathrm{e}^{-\lambda(y_1-y_2)} \beta(y_2)$$
$$\times \int_0^{y_2} \cdots \int_0^{y_{k-1}} \lambda\, \mathrm{e}^{-\lambda(y_{k-1}-y_k)} \beta(y_k)\, \mathrm{e}^{-\lambda y_k}\, \mathrm{d}y_k \cdots \mathrm{d}y_2\, \mathrm{d}y_1$$
$$= \lambda^k \mathrm{e}^{-\lambda v} \int_0^v \beta(y_1) \int_0^{y_1} \beta(y_2) \int_0^{y_2} \cdots \int_0^{y_{k-1}} \beta(y_k)\, \mathrm{d}y_k \cdots \mathrm{d}y_2\, \mathrm{d}y_1.$$

If we define

$$\gamma(v) \overset{\text{def}}{=} \int_0^v \beta(z)\, \mathrm{d}z = \int_0^v [1 - B(z)]\, \mathrm{d}z, \tag{22}$$

then we have

$$\gamma(0) = 0 \tag{23}$$

$$\int_0^v \gamma^n(z)\, \mathrm{d}\gamma(z) = \frac{\gamma^n + 1(v) - \gamma^{n+1}(0)}{n+1} = \frac{\gamma^{n+1}(v)}{n+1}. \tag{24}$$

$$P_{\text{NL}}(k, v) = \lambda^k \mathrm{e}^{-\lambda v} \int_0^v \int_0^{y_1} \cdots \int_0^{y_{k-2}} \int_0^{y_{k-1}} \mathrm{d}\gamma(y_k)\, \mathrm{d}\gamma(y_{k-1})\, \mathrm{d}\gamma(y_{k-2}) \cdots \mathrm{d}\gamma(y_1)$$
$$= \lambda^k \mathrm{e}^{-\lambda v} \int_0^v \int_0^{y_1} \cdots \int_0^{y_{k-2}} \gamma(y_{k-1})\, \mathrm{d}\gamma(y_{k-1})\, \mathrm{d}\gamma(y_{k-2}) \cdots \mathrm{d}\gamma(y_1)$$
$$= \lambda^k \mathrm{e}^{-\lambda v} \int_0^v \int_0^{y_1} \cdots \int_0^{y_{k-3}} \frac{\gamma^2(y_{k-2})}{2}\, \mathrm{d}\gamma(y_{k-2}) \cdots \mathrm{d}\gamma(y_1)$$
$$= \cdots$$

and finally

$$P_{\text{NL}}(k, v) = \frac{[\lambda\gamma(v)]^k}{k!}\, \mathrm{e}^{-\lambda v}. \tag{25}$$

For the case of a D_qM service time probability distribution we have

$$\beta(x)=\begin{cases}1, & x\leqslant q\bar{x}\\ e^{-(\mu x-q)/p}, & x>q\bar{x},\end{cases} \tag{26}$$

$$\gamma(x)=\begin{cases}x, & x\leqslant q\bar{x}\\ \dfrac{q}{\mu}+\dfrac{p}{\mu}\left[1-e^{-(\mu x-q)/p}\right], & x>q\bar{x},\end{cases} \tag{27}$$

$$P_{\mathrm{NL}}(k,v)=\begin{cases}\dfrac{(\lambda v)^k}{k!}\,e^{-\lambda v}, & v\leqslant q\bar{x}\\ \dfrac{\rho^k\left[1-p\,e^{-(\mu v-q)/p}\right]^k}{k!}\,e^{-\lambda v}, & v>q\bar{x},\end{cases} \tag{28}$$

where $\rho\overset{\text{def}}{=}\lambda\bar{x}=\lambda/\mu$. For $q=0$ (memoryless service times) equation (28) reduces to

$$P_{\mathrm{NL}}(k,v)=\frac{\rho^k(1-e^{-\mu v})^k}{k!}\,e^{-\lambda v},\quad v\geqslant 0. \tag{29}$$

For $q=1$ (deterministic service times) equation (28) reduces to

$$P_{\mathrm{NL}}(k,v)=\begin{cases}\dfrac{(\lambda v)^k}{k!}\,e^{-\lambda v}, & v\leqslant\bar{x}\\ \dfrac{\rho^k}{k!}\,e^{-\lambda v}, & v>\bar{x}.\end{cases} \tag{30}$$

The probability $P_{\mathrm{NL}}(k,v)$ above can be used for the Poisson winner queues with any service time distribution and any discipline. We gave explicit expressions for the D_qM service time distribution.

4.4. Finding $P_{\mathrm{NW}}(k, v)$

We define the probability $P_{\mathrm{NW},i}(k,v)$, $i=1,2,\ldots,k$ in the same way as $P_{\mathrm{NW}}(k,v)$ with the restriction that the ith customer is the one that wins. We can now write

$$P_{\mathrm{NW},i}(k,v)=\lambda^k e^{-\lambda v}\int_0^v\beta(y_1)\int_0^{y_1}\beta(y_2)\int_0^{y_2}\cdots\int_0^{y_{i-1}}b(y_i)\times\int_0^{y_i}\cdots\int_0^{y_{k-1}}\beta(y_k)\,\mathrm{d}y_k\cdots\mathrm{d}y_2\,\mathrm{d}y_1, \tag{31}$$

where $b(x)$ is the probability density of the service times. From this we may then find $P_{\mathrm{NW}}(k,v)$ from

$$P_{\mathrm{NW}}(k,v)=\sum_{i=1}^{k}P_{\mathrm{NW},i}(k,v). \tag{32}$$

Equations (31) and (32) are as far as we can go for a general Poisson queue. For $M/D_qM(G)$, a Poisson winner queue with any discipline, we can derive an explicit expression for $P_{\mathrm{NW}}(k,v)$ as follows. Recall that

$$b(x)=\begin{cases}0, & x\leqslant q\bar{x}\\ \dfrac{\mu}{p}\,e^{-(\mu x-q)/p}, & x>q\bar{x}.\end{cases} \tag{33}$$

From equations (26) and (33) we have the following relation between $b(x)$ and $\beta(x)$:

$$b(x)=\begin{cases}0, & x\leqslant q\bar{x}\\ \dfrac{\mu}{p}\beta(x), & x>q\bar{x}.\end{cases} \tag{34}$$

From equations (31) and (34) we see that $P_{NW,i}(k, v) = 0$ for $0 \leqslant v \leqslant q\bar{x}$. Thus, for $v > q\bar{x}$ we have

$$P_{NW,i}(k, v) = \lambda^k \, e^{-\lambda v} \frac{\mu}{p} \int_{q\bar{x}}^{v} \beta(y_1) \int_{q\bar{x}}^{y_1} \beta(y_2) \int_{q\bar{x}}^{y_2} \cdots \int_{q\bar{x}}^{y_{i-1}} \beta(y_i) \int_0^{y_i} \beta(y_{i+1}) \times \int_0^{y_{i+1}} \cdots \int_0^{y_{k-1}} \beta(y_k) \, dy_k \cdots dy_2 \, dy_1, \quad v > q\bar{x}.$$

The $k-i$ innermost integrals from the above equation may be found successively using equation (24) (in the same way we solved the k integrals for the $P_{NL}(k, v)$), to obtain:

$$P_{NW,i}(k, v) = \lambda^k \, e^{-\lambda v} \frac{\mu}{p} \int_{q\bar{x}}^{v} \beta(y_1) \int_{q\bar{x}}^{y_1} \beta(y_2) \int_{q\bar{x}}^{y_2} \cdots \int_{q\bar{x}}^{y_{i-1}} \beta(y_i) \frac{\gamma^{k-i}(y_i)}{(k-i)!} \, dy_i \cdots dy_2 \, dy_1, \quad v > q\bar{x}.$$

We now define

$$\beta_0(x) \overset{\text{def}}{=} \begin{cases} 0, & x \leqslant q\bar{x} \\ \beta(x), & x > q\bar{x}, \end{cases}$$

$$\gamma_0(x) \overset{\text{def}}{=} \int_0^x \beta_0(z) \, dz.$$

The following holds for $x > q\bar{x}$

$$\gamma(x) = \int_0^{q\bar{x}} \beta(z) \, dz + \gamma_0(x) = \gamma(q\bar{x}) + \gamma_0(x)$$

and from equation (27) we get

$$\gamma(x) = q\bar{x} + \gamma_0(x). \tag{35}$$

$P_{NW,i}(k, v)$ now becomes, for $v > q\bar{x}$

$$\begin{aligned} P_{NW,i}(k, v) &= \lambda^k \, e^{-\lambda v} \frac{\mu}{p} \int_0^{v} \int_0^{y_1} \int_0^{y_2} \cdots \int_0^{y_{i-1}} \frac{[q\bar{x} + \gamma_0(y_i)]^{k-i}}{(k-i)!} \, d\gamma_0(y_i) \cdots d\gamma_0(y_1) \\ &= \frac{\mu}{p} \frac{\lambda^k}{(k-i)!} \, e^{-\lambda v} \sum_{m=0}^{k-i} \binom{k-i}{m} (q\bar{x})^{k-i-m} \\ &\quad \times \int_0^{v} \int_0^{y_1} \int_0^{y_2} \cdots \int_0^{y_{i-1}} \gamma_0^m(y_i) \, d\gamma_0(y_i) \cdots d\gamma_0(y_1), \quad v > q\bar{x}. \end{aligned}$$

The i integrals from the above equation can be found successively using equation (24) to obtain:

$$\begin{aligned} P_{NW,i}(k, v) &= \frac{\mu}{p} \frac{\lambda^k}{(k-i)!} \, e^{-\lambda v} \sum_{m=0}^{k-i} \binom{k-i}{m} (q\bar{x})^{k-i-m} \frac{\gamma_0^{m+i}(v)}{(m+1)(m+2) \cdots (m+i)} \\ &= \frac{\mu}{p} \lambda^k \, e^{-\lambda v} \sum_{m=0}^{k-i} \frac{(q\bar{x})^{k-i-m} \gamma_0^{m+i}(v)}{(m+i)!(k-i-m)!}, \quad v > q\bar{x}. \end{aligned}$$

If we now replace $m+i$ by n, we get:

$$P_{NW,i}(k, v) = \frac{\mu}{p} \lambda^k \, e^{-\lambda v} \sum_{n=i}^{k} \frac{(q\bar{x})^{k-n} \gamma_0^n(v)}{n!(k-n)!}, \quad v > q\bar{x}, \tag{36}$$

which gives us

$$P_{NW,i}(k, v) = \frac{\mu}{p} \frac{\lambda^k}{k!} \, e^{-\lambda v} \sum_{n=i}^{k} \binom{k}{n} (q\bar{x})^{k-n} \gamma_0^n(v), \quad v > q\bar{x}.$$

Using equation (32) we find $P_{NW}(k, v)$ as follows.

$$\begin{aligned}
P_{NW}(k, v) &= \sum_{i=1}^{k} \frac{\mu}{p} \frac{\lambda^k}{k!} e^{-\lambda v} \sum_{n=i}^{k} \binom{k}{n} (q\bar{x})^{k-n} \gamma_0^n(v) \\
&= \frac{\mu}{p} \frac{\lambda^k}{k!} e^{-\lambda v} \sum_{n=1}^{k} \sum_{i=1}^{n} \binom{k}{n} (q\bar{x})^{k-n} \gamma_0^n(v) \\
&= \frac{\mu}{p} \frac{\lambda^k}{k!} e^{-\lambda v} \sum_{n=0}^{k} n \binom{k}{n} (q\bar{x})^{k-n} \gamma_0^n(v) \\
&= \frac{\mu}{p} \frac{\lambda^k}{k!} e^{-\lambda v} \gamma_0(v) \frac{d}{d\gamma_0(v)} \left[q\bar{x} + \gamma_0(v) \right]^k, \quad v > q\bar{x}.
\end{aligned}$$

Thus,

$$P_{NW}(k, v) = \frac{\mu}{p} \lambda^k \gamma_0(v) \frac{\gamma^{k-1}(v)}{(k-1)!} e^{-\lambda v}, \quad v > q\bar{x}. \tag{37}$$

Finally, using the expressions for $\gamma(v)$ and $\gamma_0(v)$ from equations (27) and (35), we get

$$P_{NW}(k, v) = \begin{cases} \lambda \left[1 - e^{-(\mu v - q)/p}\right] \dfrac{\rho^{k-1}\left[1 - p\, e^{-(\mu v - q)/p}\right]^{k-1}}{(k-1)!} e^{-\lambda v}, & v > q\bar{x},\ k \geqslant 1 \\ 0, & \text{otherwise.} \end{cases} \tag{38}$$

For memoryless service times ($q = 0$), equation (38) becomes

$$P_{NW}(k, v) = \begin{cases} \lambda \dfrac{\rho^{k-1}(1 - e^{-\mu v})^k}{(k-1)!} e^{-\lambda v}, & v \geqslant 0,\ k \geqslant 1 \\ 0, & \text{otherwise.} \end{cases} \tag{39}$$

For deterministic service times ($q = 1$), equation (38) becomes

$$P_{NW}(k, v) = \begin{cases} \lambda \dfrac{\rho^{k-1}}{(k-1)!} e^{-\lambda v}, & v > \bar{x},\ k \geqslant 1 \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

The probability $P_{NW}(k, v)$ above can be used for Poisson winner queues with D_qM service time distribution and any discipline. For other service time probability distributions it may also be possible to find $P_{NW}(k, v)$, starting from equations (31) and (32), and using a technique similar to the one used here.

5. Applications

5.1. M/M(SR): exact $p_{i,j}$s

As our first application of the results from Section 4, we study the M/M(SR) queue. We substitute equations (17), (18), (29) and (39) into equation (9) to get the following expressions for M/M(SR).

$$p_{i,j}(v) = \begin{cases} \mu \dfrac{\left[\rho(1 - e^{-\mu v})\right]^{j+1}}{j!} e^{-\lambda v}, & i = 0,\ j \geqslant 0,\ v \geqslant 0 \\ \mu\left[(j+1)\mu v + j - i + 1\right](1 + \mu v)^{i-1} \dfrac{\left[\rho(1 - e^{-\mu v})\right]^{j-i+1}}{(j-i+1)!} e^{-(\lambda + i\mu)v}, & \\ \qquad i \geqslant 1,\ j \geqslant i - 1,\ v \geqslant 0 & \\ 0, & \text{otherwise,} \end{cases} \tag{41}$$

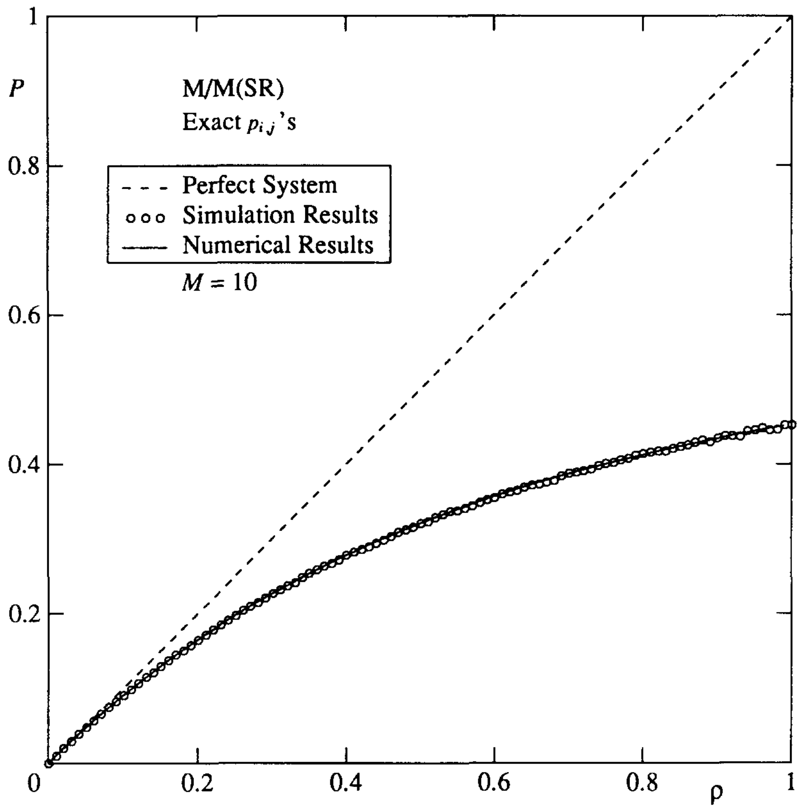

Fig. 9. Normalized power for M/M(SR).

which, after integration according to equation (8), gives

$$p_{i,j} = \begin{cases} \rho^{j+1} \sum_{m=0}^{j+1} \frac{(-1)^m}{m!(j+1-m)!} \frac{j+1}{\rho+m}, & i=0,\ j \geqslant 0 \\ \rho^{j-i+1} \sum_{m=0}^{j-i+1} \frac{(-1)^m}{m!(j-i+1-m)!} \sum_{k=0}^{i-1} \frac{(i-1)!}{(i-1-k)!} \frac{1}{(\rho+i+m)^{k+2}} & \\ \quad \times \left[(j+1)(k+1) + (\rho+i+m)(j-i+1)\right], & i \geqslant 1,\ j \geqslant i-1 \\ 0, & \text{otherwise.} \end{cases} \tag{42}$$

Following the numerical calculation procedure depicted in Section 4, we obtain results for the normalized power P and normalized response time T_n which we plot versus the load ρ in Figs. 9 and 10. In the same figures we also show simulation results for M/M(SR). The numerical results coincide so well with the simulation results that the two curves are indistinguishable. Moreover, we show the behavior of the "perfect" system defined as one with $T_n = 1$ and $P = \rho$. For high ρ, power for M/M(SR) seems to be approaching a constant. In fact, we obtain finite response time for all $\rho < \infty$ in this system. (See, for example, Fig. 11). This unusual behavior is due to the fact that the winner among a group of customers will be that customer who finishes service first; thus the mean service time of successful customers is smaller than $1/\mu$, and, in fact, will approach zero as $\rho \to \infty$. Those customers with "large" service times will be killed and allowed to reselect their service times until they are successful. Figure 11 shows numerical calculations for different values of the precision parameter M and the dotted curve is our estimate of the power shown approaching a constant value. The explanation for this fact, i.e., that the power does not fall off at high load (even for $\rho \gg 1$), is that *successful* service times approach zero for high ρ.

A two-dimensional Markov chain used to model M/M(SR) is given in [4].

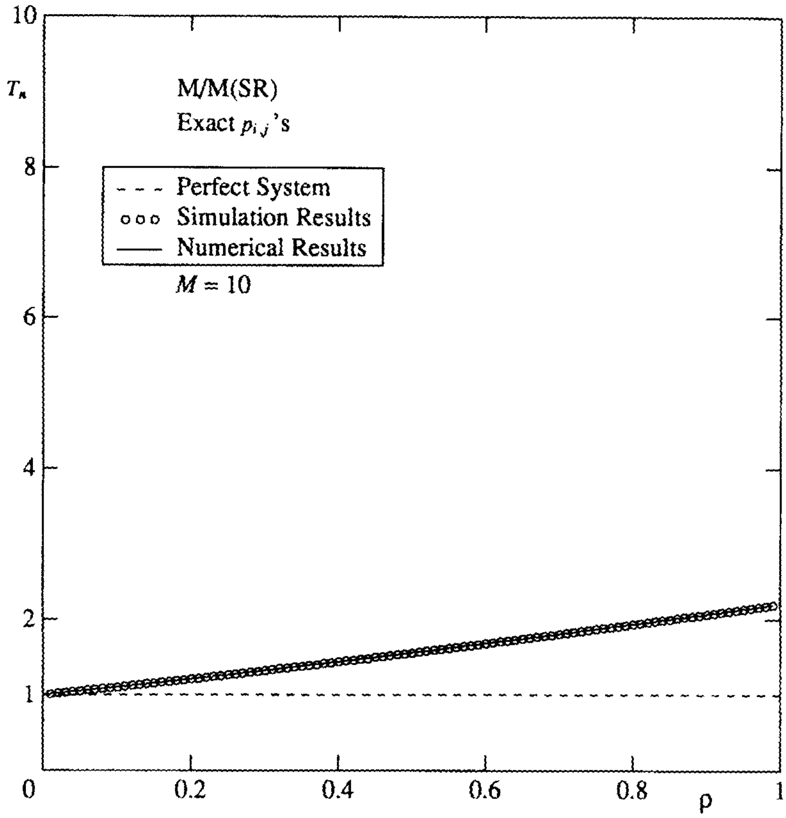

Fig. 10. Normalized average response time for M/M(SR).

5.2. *M/D(S): approximate* $p_{i,j}$s

A second application is to the M/D(S) queue. We substitute equations (20), (21), (30) and (40) into equation (9) to get the following expressions for M/D(S):

$$p_{i,j}(v) = \begin{cases} \lambda \dfrac{\rho^j}{j!}\, e^{-\lambda v}, & i = 0,\ j \geqslant 0,\ v \geqslant \bar{x} \\ \mu\left[(j+1)\, e^{1-\mu v} - (j-i+1)/e\right] \dfrac{\left(e^{1-\mu v} - 1/e\right)^{i-1}}{(1-1/e)^i}\, \dfrac{\rho^{j-i+1}}{(j-i+1)!}\, e^{-\lambda v}, & \\ \qquad i \geqslant 1,\ j \geqslant i-1,\ \bar{x} < v \leqslant 2\bar{x} & \\ 0, & \text{otherwise,} \end{cases} \tag{43}$$

which, after integration according to equation (8), gives

$$p_{i,j} = \begin{cases} \dfrac{\rho^j}{j!}\, e^{-\rho}, & i = 0,\ j \geqslant 0 \\ \dfrac{\rho^{j-i+1}}{(j-i+1)!}\, \dfrac{e^{-(\rho+1)}}{(1-/e)^i} \displaystyle\sum_{k=0}^{i-1} \binom{i-1}{k} (-1/e)^{i-1-k} & \\ \quad \times \left\{ \left[e - e^{-(\rho+k)}\right] \dfrac{j+1}{\rho+k+1} - \left[1 - e^{-(\rho+k)}\right] \dfrac{j-i+1}{\rho+k} \right\}, & i \geqslant 1,\ j \geqslant i-1 \\ 0, & \text{otherwise.} \end{cases} \tag{44}$$

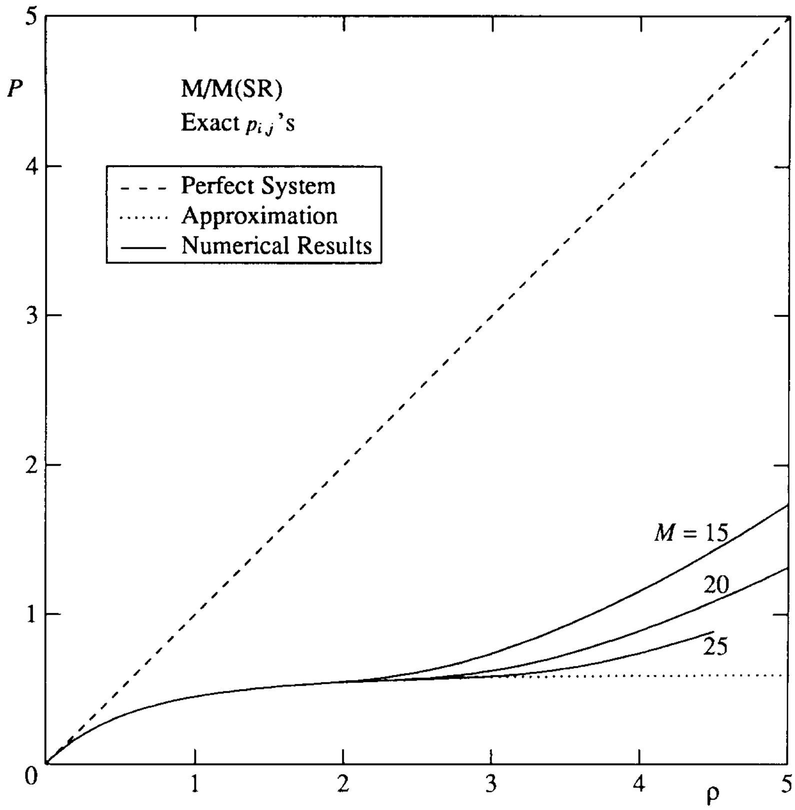

Fig. 11. Normalized power for M/M(SR) with high ρ.

Following the numerical calculation procedure depicted in Section 4, we plot the normalized power P and normalized response time T_n versus the load ρ in Figs. 12 and 13. In the same figures we also show the simulation results for M/D(S) given previously (M/D(SR) is equivalent to the M/D(SN) due to the deterministic service times).

Note the two tails of the power plotted in Fig. 12. The lower tail is the power calculated with higher precision, i.e., M is higher. These tails are due to errors caused by the numerical calculations. The exact power for M/D(S) should drop to zero for $\rho = 1$. It is interesting that the shape of the normalized power for the queue M/M/1, given as $P_{M/M/1} = \rho(1-\rho)$! At this point we conjecture, but have not proven, that it actually *is the same* as for M/M/1.

5.3. $M/D_qM(BR)$: exact $p_{i,j}$s

Our last application is to the M/D_qM(BR) queue. We substitute equations (14), (15), (28) and (38) into equation (9) to get the following expressions for M/D_qM(BR).

$$p_{i,j}(v) = \begin{cases} \lambda\left[1 - e^{-(\mu v - q)/p}\right] \dfrac{\rho^j\left[1 - p\, e^{-(\mu v - q)/p}\right]^j}{j!}\, e^{-\lambda v}, & v \geqslant q\bar{x}, \quad i = 0,\ j \geqslant 0 \\ e^{-i(\mu v - q)/p} \dfrac{\lambda}{j-i+1} \dfrac{\rho^{j-i}\left[1 - p\, e^{-(\mu v - q)/p}\right]^{j-i}}{(j-i)!} & \\ \quad \times \left\{ iq/p + (j+1)\left[1 - e^{-(\mu v - q)/p}\right] \right\} e^{-\lambda v}, & v \geqslant q\bar{x},\ i \geqslant 1,\ j \geqslant i-1 \\ 0, & \text{otherwise,} \end{cases} \tag{45}$$

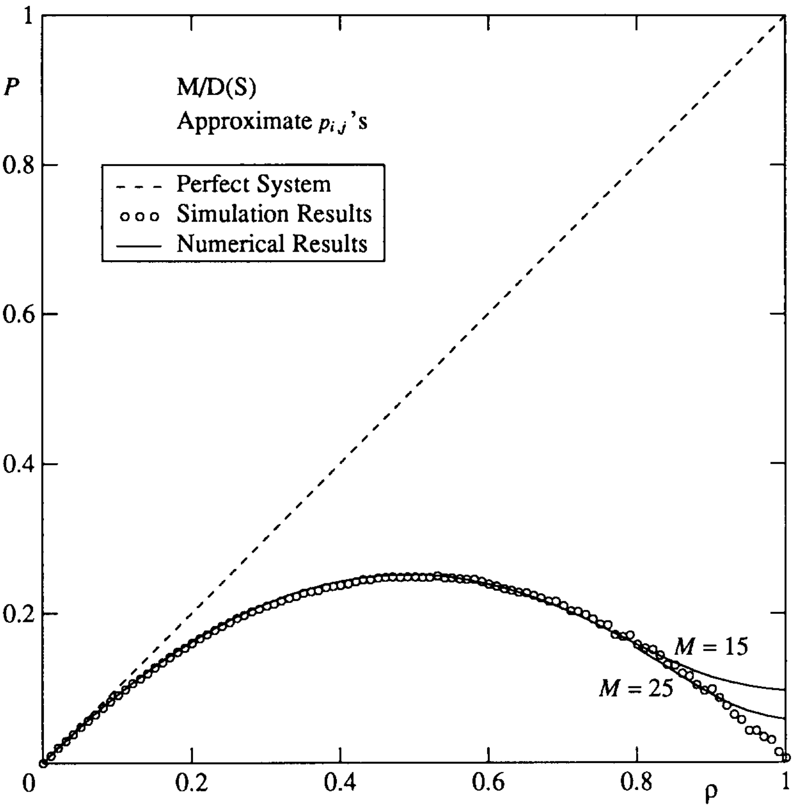

Fig. 12. Normalized power for M/D(S).

which, after integration according to equation (8), gives

$$p_{i,j} = \begin{cases} \rho^{j+1} \mathrm{e}^{-q\rho} \sum_{m=0}^{j} \dfrac{(-p)^m}{m!(j-m)!} \left[\dfrac{1}{\rho + m/p} - \dfrac{1}{\rho + (m+1)/p} \right], \\ \qquad\qquad\qquad\qquad i = 0,\ j \geqslant 0 \\ \dfrac{\rho^{j-i+1}}{j-i+1} \mathrm{e}^{-q\rho} \sum_{m=0}^{j-i} \dfrac{(-p)^m}{m!(j-i-m)!} \left[\dfrac{j+1+iq/p}{\rho + (i+m)/p} - \dfrac{j+1}{\rho + (i+m+1)/p} \right], \\ \qquad\qquad\qquad\qquad i \geqslant 1,\ j \geqslant i-1 \\ 0, \qquad\qquad\qquad\quad \text{otherwise}. \end{cases} \tag{46}$$

Following the numerical calculation procedure depicted in Section 4, we plot the normalized power P and normalized response time T_n versus the load ρ in Figs. 14 and 15. In the same figures we also show the simulation results for $\mathrm{M/D}_q\mathrm{M(BR)}$.

Let us now consider M/M(BR) by setting $q = 0$. When there are k customers in the system, the rate out is $k\mu$, while the rate in is always λ. This is also the behavior of $\mathrm{M/M/\infty}$. Indeed, M/M(BR) *is equivalent* to $\mathrm{M/M/\infty}$, which is why Fig. 14 shows that M/M(BR) gives "perfect" performance, that is,

$$P = \rho. \tag{47}$$

When we set $q = 1$ we get M/D(B), which is simply an ordinary M/D/1 queue. Thus we get the same normalized power as for M/D/1. In M/D(B), service time is never wasted (i.e., the service for exactly one customer is always useful, given at least one customer in the system). As soon as a customer leaves, the

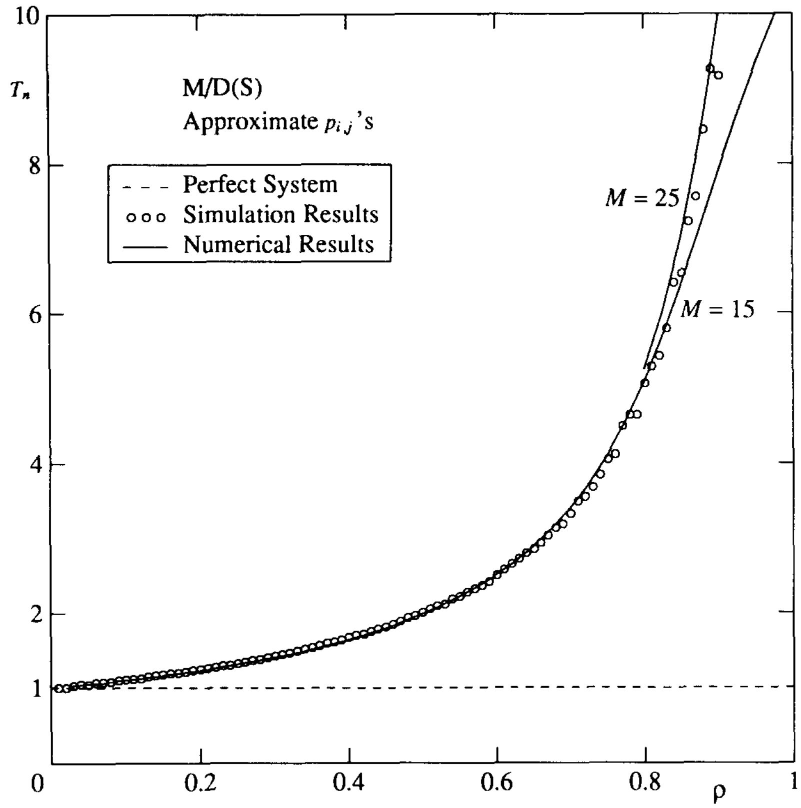

Fig. 13. Normalized average response time for M/D(S).

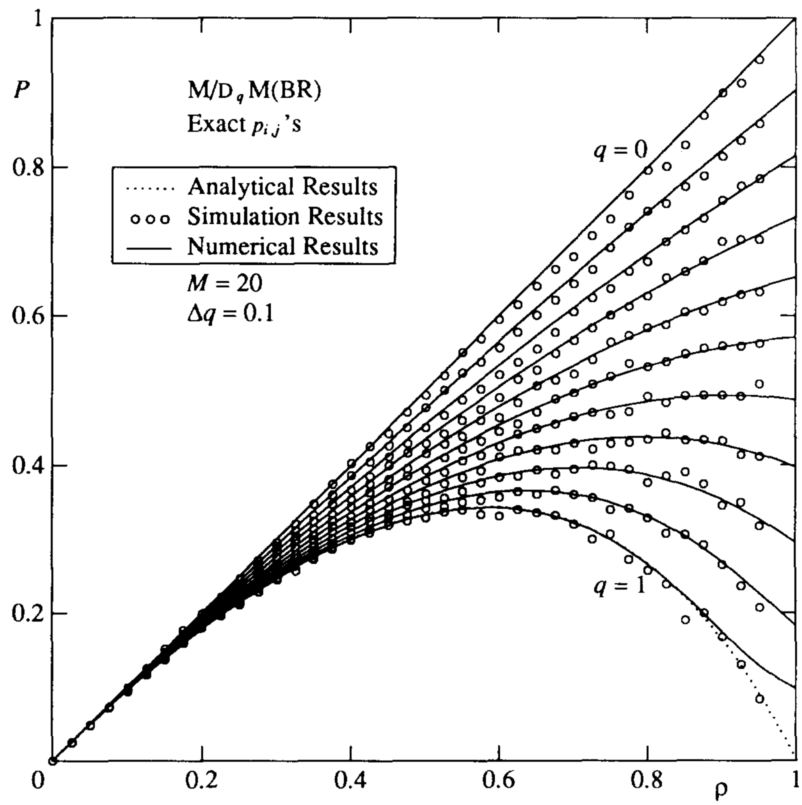

Fig. 14. Normalized power for $\mathrm{M/D}_q\mathrm{M(BR)}$.

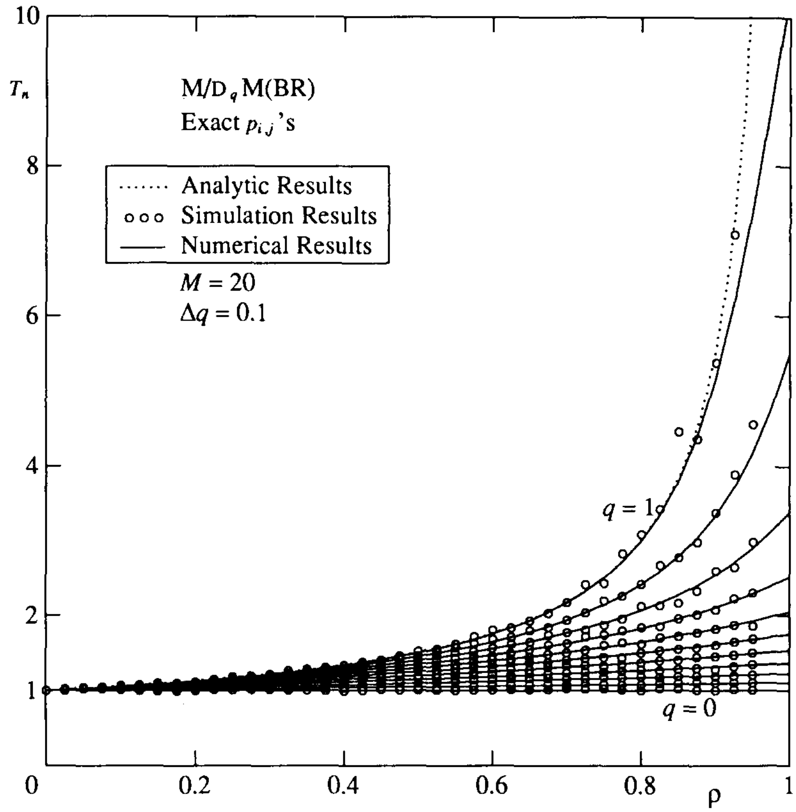

Fig. 15. Normalized average response time for M/D$_q$M(BR).

customers left in the system will restart. It is obvious that the system behaves as an M/D/1 queue. Thus we get

$$P = 2\rho(1-\rho)/(2-\rho) \tag{48}$$

for $q = 1$, and this is plotted as the dashed curve in Fig. 14.

Figures 16 and 17 show the normalized power and the average response system time, respectively, for M/D$_q$M(BR) as functions of ρ and q.

6. Conclusion

We have studied special types of queues, which we call winner queues. One obvious application of winner queues is the performance evaluation of optimistic concurrency control schemes in databases. The winner queues studied are special cases of the winner queues considered in [4].

For these systems we investigated the average system time of customers. The results obtained by simulation and analysis were shown in terms of the normalized average system time and the normalized power. The analysis also gave the distribution of the number of customers in the system.

We obtained simulation results for four different classes of winner queues: silent-redraw (SR), silent-noredraw (SN), broadcast-redraw (BR), and broadcast-noredraw (BN). The results showed power curves with q varying from 0 to 1. We found that redraw queues perform better than noredraw queues, broadcast queues perform better than silent queues, and that queues with smaller q perform better for redraw queues, while for noredraw queues higher q gives better results. These observations are summarized in Table 1.

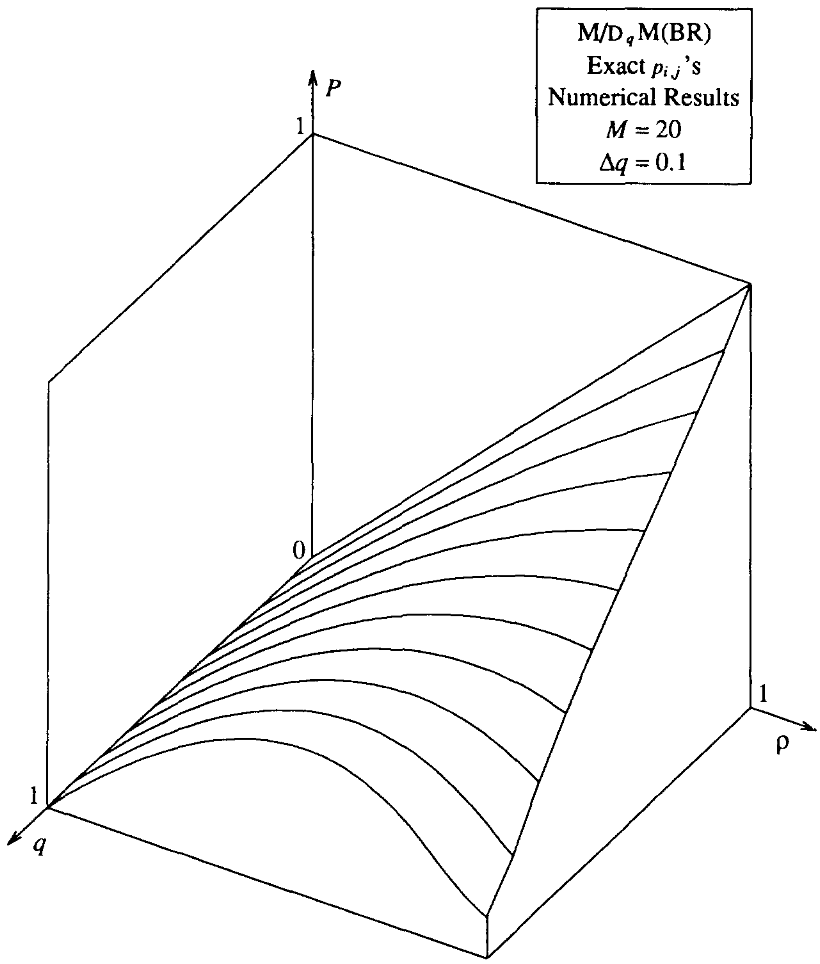

Fig. 16. Normalized power in 3-D for M/D$_q$M(BR).

Table 1
Effect of system parameters to the performance

Systems compared	Better	Worse
Silent		√
vs.		
Broadcast	√	
Redraw	√	
vs.		
Noredraw		√
Redraw/Memoryless	√	
vs.		
Redraw/Deterministic		√
Noredraw/Memoryless		√
vs.		
Noredraw/Deterministic	√	

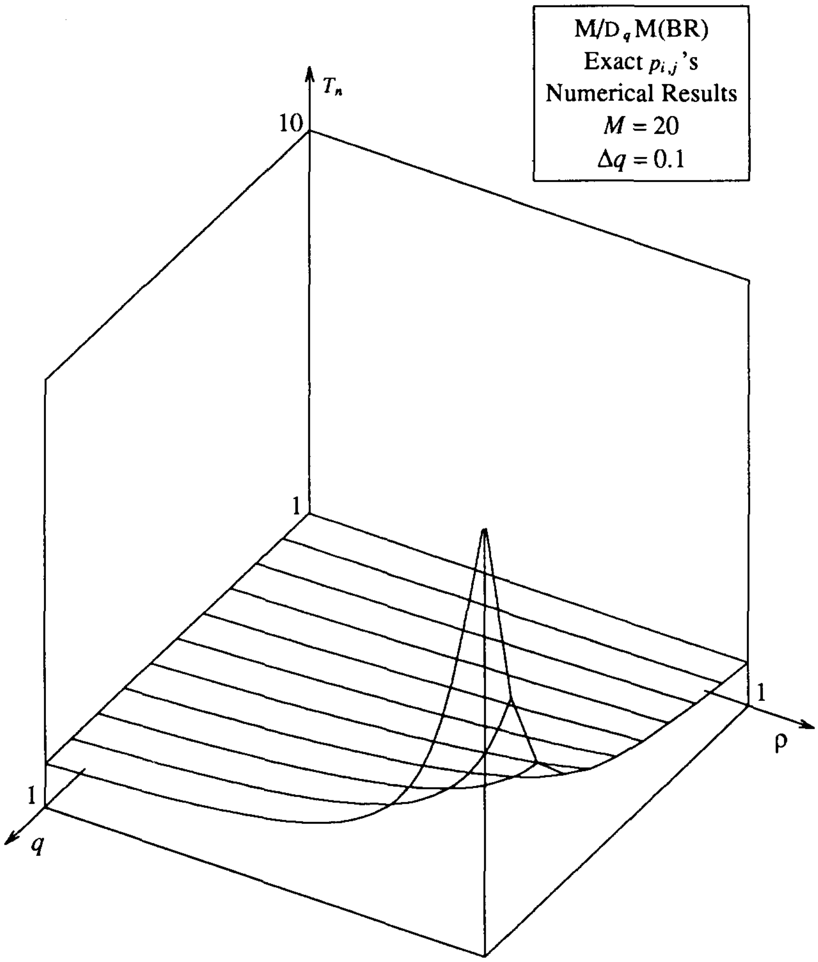

Fig. 17. Normalized average response time in 3-D for M/D$_q$M(BR).

We obtained numerical results using exact expressions for the transition probabilities for the winner queues M/M(SR) and M/D$_q$M(BR). Using approximate expressions for the transition probabilities, we numerically solved the winner queue M/D(S). Analytic results were found for the winner queues M/M(BR) and M/D(B). Figure 18 gives an overview of these results.

More general winner queues, in which restarts differ from the original service time distribution, are analyzed in [4]. In [4] we consider cases in which concurrent customers do not necessarily conflict. These cases correspond to transaction processing in database systems.

We know that both numerical calculations and analytic expressions for M/M(BR) and M/D(B) in Section 5.3 are valid. Suppose that we could mathematically derive the analytic expressions given in equations (47) and (48) from the transition probabilities given in equation (46). Then, we might also be able to find analytic expressions for the winner queues M/M(SR), M/D(B), and M/D$_q$M(BR), $0 < q < 1$, from the transition probabilities given in equations (42), (44) and (46), respectively. This mathematical derivation is an area of further research. Further studies of noredraw winner queues would also be fruitful.

	M	D_qM	D
SR	$\boxed{E}$		$\boxed{A}$
SN			$\boxed{A}$
BR	E	$\boxed{E}$	E
BN			E

$\boxed{}$	numerical results
$\bigcirc$	analytic results
E	exact transition probabilities
A	approximate transition probs.

Fig. 18. Poisson winner queue results – overview table.

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