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A NETWORK ALGEBRA FOR THE PERFORMANCE EVALUATION OF INTERCONNECTED COMPUTER NETWORKS*

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Abstract

In this paper we develop a network algebra to evaluate the throughput-delay performance of interconnected computer networks. We start by developing a simplified analytical model for an isolated computer network in which messages are flow-controlled with a window mechanism. Using this model, we then define the notion of equivalent nets and characterize the equivalent network for systems composed of series and/or parallel connections of computer nets.

1. Introduction

One of the most important outstanding problems in computer network design is that of flow control. A flow control procedure is simply a procedure in which the flow of data across the boundary of a computer network is throttled so as to maintain the load on the network in an acceptable range. These controls may be applied by using such mechanisms as: *tokens,* which give permission for messages to flow; a *rate* at which a given flow may proceed; a *stop-and-go* procedure which turns a flow on and off according to some criteria; the introduction of *delay,* so as to slow down the flow, etc. It is not hard to see why flow control procedures are necessary in computer networks (and in many other systems of interest). Indeed, modern computer networks provide the opportunity as well as the obligation to connect different speed devices to each other through the network. It is then clear that whereas a very slow speed teletype may be connected through the network to an enormously high speed, large scale, expensive computer, it could be extremely inefficient were the teletype to interrupt the main frame of that machine every time it passed a character across the network. On the other hand, one cannot permit the memory channel from the large machine to pour megabytes per second at the 30 character per second teletype. Furthermore, one does not wish to use the network as a storage device either. Thus we see that one purpose of a flow control procedure is to protect devices from each other as well as the network from uncontrolled devices. More generally, however, networks tend to exhibit what is known as "congestion-prone" behavior [1]; this is the phenomenon whereby the throughput will increase with the applied load up to some optimum value, beyond which more load causes a reduction in throughput. The flow control procedure attempts to maintain the network load at this optimum value- a difficult control problem.

To date, little work has been done in evaluating the performance of flow control procedures in terms of their efficiency, freedom from deadlocks and degradations, and general performance profiles. These distributed dynamic control procedures tend to be extremely difficult to analyze. Moreover, the design of such procedures is extremely difficult and one may easily fall into the trap of designing highly inefficient and/or highly dangerous flow control procedures. In view of all this, the recent work in the field have been extremely welcome [2, 3, 4, 5, 6 and the references therein].

Another very important area for computer communications is the field of internetting [7]. We already see various computer networks connected to each other across the world. This interconnection of networks is proliferating faster than the growth of individual national networks themselves. The development of standards for internetting and for analytic and design procedures for such large connected networks is in its infancy; here again, the issue of flow control becomes important.

In this paper, we make a contribution to the field of flow control procedures as applied to interconnected networks. Our approach is to develop a set of analytic tools which allow certain simple interconnected network configurations to be evaluated. In particular, we focus on two simple topologies: first, we study the case of tandem networks in which data flows serially from network A to network B to network C, etc., on the way to its destination; second, we study the case of parallel connections of networks in which the packet flow may choose one of many parallel paths in passing across a geographical region. We represent each of the individual networks in an extremely simple fashion, accounting for three network parameters: the network capacity; the number of hops in the networks; and the window size which controls the flow in that network. In terms of these parameters, we then use simple statistical assumptions to evaluate the delay-throughput performance of a network in isolation. With this model, we then develop a simple network algebra, which allows us to represent series connections of many networks as an equivalent network in isolation whose three parameters are functions of each of the tandem networks' parameters; this is a convenient network collapse transformation. We also consider parallel connections of networks and here again we find that in an important special case, the parallel set can be reduced to a single network. The results reported here suggest that series-parallel networks can be reduced to single networks whose properties we understand from previous studies. This is only the beginning, and more interesting topological configurations must be studied in order to complete the algebra.

Let us now begin with a model of a single network using the window mechanism for flow control.

2. A Model for Flow Control With a Window Mechanism

In this section we develop a very simple analytic model for a network in which messages are flow-controlled with a window mechanism.

With a window mechanism, the total number of messages in the network between a source -destination pair is restricted to a maximum value. We model the network as in Fig. 1, in which the traffic controller (TC) is responsible for keeping the number of unacknowledged messages below w, the window limit. Thus we may think of a circulating pool of w "tokens", each one of which

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permits exactly one message to be sent toward the destination node; a copy of each message is retained in the buffer space provided by the TC. When the TC receives an acknowledgement (ACK) for a message, it discards its stored copy of the message and the token is returned back into the pool. This token may then be used to accept a new message from the source. Thus, we allow up to w outstanding messages between a source-destination pair. In the destination node, messages which are received are transmitted to the receiver and ACKs for them are sent back to the TC. We also assume that the traffic on the path is symmetric (i.e., there is the same flow rate of data from the receiver to the sender) and that the ACKs are piggybacked on the messages, hence there is no extra traffic due to ACKs. Further, we make a "heavy traffic" assumption; i.e., we assume that the sender is fast compared to the network so that whenever the TC can accept a new message, the sender has one ready for transmission. With these assumptions, the throughput is mainly determined by the network and throughout our study we will be concerned with the maximum throughput-delay behavior of the network within the boundaries shown in Fig. 1. Maximum throughput is the maximum rate at which the network can accept new messages (hence it is the upper bound of the accepted input rate to the network). For simplicity, when there is no ambiguity, we use "throughput" to indicate "maximum throughput."

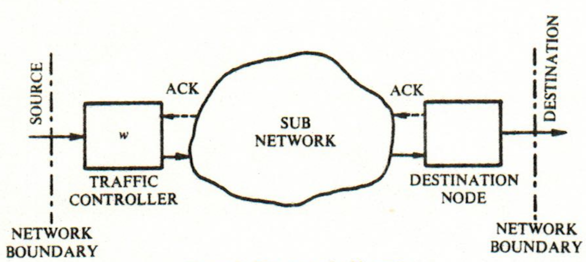


Figure 1- Structure of a Network.

The subnetwork is viewed as a source of delay and is assumed to be reliable (i.e. the network channels are noiseless). To calculate the network delay, we make the simplifying assumption that the average network delay is modeled as an n_h-hop network with each hop modeled as an $M/M/1$ queue. Since the network is assumed to be reliable, for each message sent out of the TC, an ACK is received (with probability 1); moreover, the average ACK delay (T_a) is equal to the round-trip delay and we have $T_a = 2T$, where T is the average network delay (i.e., the one-way delay). With these assumptions the average delay at each node is $1/(\gamma - \lambda)$ seconds, where λ is the traffic on network channels and γ is the capacity of each network channel, both measured in msg/sec. The average network delay is therefore

$$T = \frac{n_h}{\gamma - \lambda} \tag{1}$$

and the ACK delay is

$$T_a = 2T = \frac{2n_h}{\gamma - \lambda} \tag{2}$$

When the flow is controlled by the window mechanism described above, w messages pass through the network on the average every T_a seconds; therefore, the (maximum) throughput of the network, λ^*, is given by

$$\lambda^* = \frac{w}{T_a^*} \tag{3}$$

where T_a^* is the ACK delay when the flow controlled throughput is maximum (with window size w) and is given by Eq. (2) (when $\lambda = \lambda^*$). Solving Eqs. (2) and (3) for λ^*, we get

$$\lambda^* = \frac{w}{w + 2n_h}\gamma \tag{4}$$

Using the value of $\lambda = \lambda^*$ in Eq. (1), we get the following expression for the (maximum) network delay when the window size is w:

$$T^* = \begin{cases} \dfrac{w + 2n_h}{2\gamma} & w>0 \text{ and } \gamma>0 \\ 0 & w=0 \text{ or } \gamma=0 \end{cases} \tag{5}$$

Note that when $w = 0$ or $\gamma = 0$, there is no traffic, hence the delay is *defined* to be zero.

Eqs. (4) and (5) are the basic expressions in our analysis for the maximum network throughput and delay as a function of network channel capacity, γ, the window size, w, and the number of hops, n_h. (For a more elaborate analysis of flow control the interested reader is referred to [5].)

Equations (4) and (5) can be written in the following forms

$$\lambda^* = \frac{l^*}{l^* + 2}\gamma \tag{6}$$

and

$$T^* = \begin{cases} \dfrac{l^* + 2}{2\gamma} n_h & l^*>0 \text{ and } \gamma>0 \\ 0 & l^*=0 \text{ or } \gamma=0 \end{cases} \tag{7}$$

where $l^* = w/n_h$.

The parameter l^*, as we shall see shortly, plays an important role in our study; let us comment on its physical significance. The window size w, as used in our analysis, is the average number of messages outstanding between our given source-destination pair in the network (note, therefore, that in our model the average number of messages, rather than the actual number of messages, is kept limited). There are n_h hops (or nodes) on the path (hence there are $2n_h$ hops on a round-trip); therefore, when the network carries a traffic determined by Eq. (6), $l^* = w/n_h$ will be the average number of messages in a node, of which $l^*/2$ are being transmitted to the destination and the other half are carrying acknowledgements back to the source node (recall that we have assumed acknowledgements are piggybacked on messages sent from the destination node to the source node). In queueing theory the average number of customers in a system may be considered to be the system load [1]; therefore, $l^* = w/n_h$ is a measure of the load (or the load factor) on a node (and $l^*/2$ is the load of a channel) on the path due to the maximum traffic from the source to the destination. Note that through the window control, the load on a node is limited to w/n_h and Eq. (6) determines the traffic which generates this load; hence λ^* is the throughput capacity of the network. If, due to some limitations (which we discuss shortly), the throughput (λ) is less than λ^* given by Eq. (6), then the load on the nodes is simply

$$l = \frac{2\lambda}{\gamma - \lambda} \tag{8}$$

On the other hand if the load of the nodes is known (*say* $l < l^*$), then the throughput and delay of the network will be

$$\lambda = \frac{l}{l + 2}\gamma \qquad \text{for } 0 \leq l \leq l^* \tag{9}$$

and

$$T = \begin{cases} \dfrac{l + 2}{2\gamma} n_h & 0<l<l^* \text{ and } \gamma>0 \\ 0 & l=0 \text{ or } \gamma=0 \end{cases} \tag{10}$$

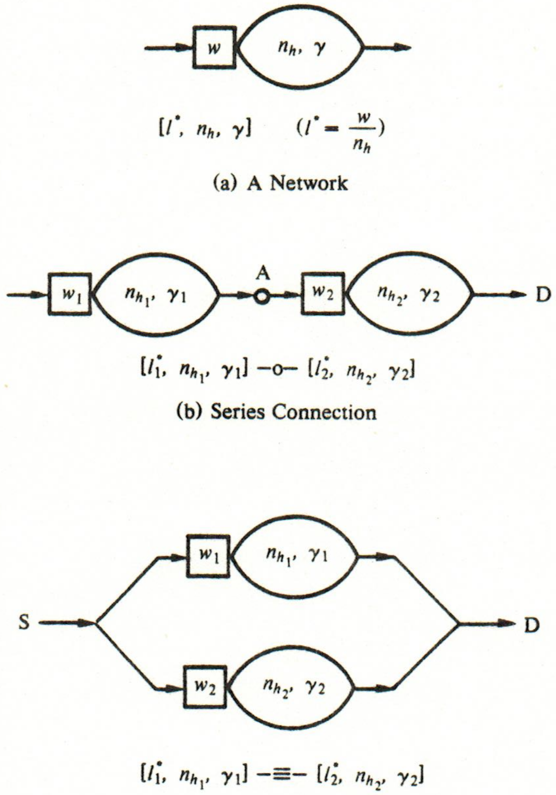
In the sequel we refer to $l^*=w/n_h$ as the *maximum load factor;* to λ^* as the *traffic handling capacity;* and to T^* as the *maximum delay* of a network (when there is no ambiguity we refer to λ^* and T^* simply by throughput and delay). Note that the load factor (l) is different from the utilization factor ρ ($=\lambda/\gamma$) defined to be the fraction of the channel capacity used for transmission.

Before proceeding further, we find it useful to comment on the accuracy of the analysis in view of the simplifying assumptions which led to Eqs. (4) and (5). In the Appendix an exact analysis of the window mechanism flow control is presented, where we find the following exact expressions for the (maximum) throughput and delay

$$\lambda^* = \frac{w}{w + 2n_h - 1}\gamma = \frac{l^*}{l^* + 2 - 1/n_h} \tag{11}$$

$$T^* = \frac{w + 2n_h - 1}{2\gamma} = \frac{l^* + 2 - 1/n_h}{2\gamma} n_h \tag{12}$$

$$w>0 \text{ (or } l^*>0\text{) and } \gamma>0$$



(a) A Network

(b) Series Connection

(c) Parallel Connection

Figure 2- Short Hand Notation.

These expressions seem simple enough to be used in our later analysis; however, the term $1/n_h$ in Eqs. (11) and (12) causes the analysis to become somewhat more complex. Considering the fact that for $n_h >> 1$ the term $1/n_h$ becomes negligible compared to 2, and that Eqs. (4) and (5) underestimate the performance of the network, we take the liberty of using these simplified expressions for the (maximum) throughput and delay of the network.

Eqs. (6) and (7) show that a network (for the purpose of studying its throughput and delay) can be totally characterized by three parameters: γ, the maximum transmission rate of the channels; n_h, the number of hops along the path; and l^* ($=w/n_h$), the maximum load factor of nodes as defined earlier. Symbolically we designate a network by a triple $[l^*, n_h, \gamma]$ (Fig. 2-a). In this figure the box with a 'w' inside, indicates a TC with window size w in front of a network with path length 'n_h' and transmission rate γ msg/sec. In what follows we seek to find an isolated equivalent network, $[l_e^*, n_{h_e}, \gamma_e]$, whose behavior is the same as a system composed of series and/or parallel connections of networks each of which will be represented as in Fig. 2-a. The symbols '—o—' and '—≡—' will be used to indicate series and parallel connections of two networks, respectively (Fig. 2-b and 2-c); the equivalence relationship will be indicated by the symbol ≡. Throughout this paper we assume that $w>0$ and $\gamma>0$. We begin by defining equivalent networks.

Definition

Two networks are said to be equivalent if the traffic handling capacity and the maximum delay of both are identical.

3. Series Connections of Networks

Consider Fig. 2-b in which two networks are connected together in series (the connecting node A is usually referred to as the GATEWAY [7]). Messages from source S, connected to network 1, are to be transmitted to destination D, connected to network 2. We assume that the operation of the two networks are independent of each other. That is, when a message from network 1 is delivered to network 2 (via the GATEWAY), the ACK for this message is sent back to the TC of network 1; a copy of this message is kept by the TC of the second network until the ACK is received from the destination node. The only interaction between the two networks arises from their throughput handling limitations. We start with the analysis of two networks connected in series and then generalize the results for the case of more than two nets.

Consider two networks connected to each other in series; the traffic handling capacity and the maximum delay of each network are given by Eqs. (6) and (7). Depending if $\lambda_2^* = \lambda_1^*$, $\lambda_2^* > \lambda_1^*$, or $\lambda_2^* < \lambda_1^*$, we consider three cases.

Case I: $\lambda_2^* = \lambda_1^*$

When the traffic handling capacity of the two networks are equal, we have

$$\frac{l_1^*}{l_1^* + 2}\gamma_1 = \frac{l_2^*}{l_2^* + 2}\gamma_2$$

Solving the above equation for l_i^*, i=1, 2, we find that the maximum load factors of the two networks are related to each other as follows:

$$l_i^* = \frac{2l_j^*\gamma_j}{l_j^*(\gamma_i - \gamma_j) + 2\gamma_i} \qquad i,j=1 \text{ or } 2 \text{ and } i \neq j$$

The throughput of the system is therefore

$$\lambda_e^* = \lambda_i^* = \frac{l_i^*}{l_i^* + 2}\gamma_i \qquad i=1 \text{ or } 2 \tag{13}$$

The maximum system delay is the sum of the delays in the two networks, i.e.,

$$T_e^* = T_1^* + T_2^*$$

or

$$T_e^* = \frac{l_i^* + 2}{2\gamma_i}\left[1 + \frac{2}{l_i^*(\frac{\gamma_j}{\gamma_i} - 1) + 2\frac{\gamma_j}{\gamma_i}}\frac{n_{h_j}}{n_{h_i}}\right] n_{h_i} \tag{14}$$

$$i, j=1 \text{ or } 2 \text{ and } i \neq j$$

Comparing Eqs. (13) and (14) with Eqs. (6) and (7), for $\lambda_1^* = \lambda_2^*$ we have

$$[l_1^*, n_{h_1}, \gamma_1] -o- [l_2^*, n_{h_2}, \gamma_2] \equiv [l_e^*, n_{h_e}, \gamma_e]$$

where

$$l_e^* = l_i^* \tag{15-a}$$

$$n_{h_e} = \left[1 + \frac{2}{l_i^*(\frac{\gamma_j}{\gamma_i} - 1) + 2\frac{\gamma_j}{\gamma_i}} \frac{n_{h_j}}{n_{h_i}} \right] n_{h_i} \qquad j \neq i \tag{15-b}$$

$$\gamma_e = \gamma_i \tag{15-c}$$

and the equivalent window size will be

$$w_e = l_e^* n_{h_e} = \left[1 + \frac{2}{l_i^*(\frac{\gamma_j}{\gamma_i} - 1) + 2\frac{\gamma_j}{\gamma_i}} \frac{n_{h_j}}{n_{h_i}} \right] w_i \qquad j \neq i \tag{15-d}$$

In Eqs. (15-a) to (15-d) i can be either 1 or 2.

Case II: $\lambda_2^* > \lambda_1^*$

In this case network 1 is the bottleneck; hence we have

$$\lambda_e^* = \lambda_1^* = \frac{l_1^*}{l_1^* + 2} \gamma_1 \tag{16}$$

When network 2 carries a traffic equal to λ_1^*, the load at its nodes becomes (Eq. (8))

$$l_2 = \frac{2 l_1^* \gamma_1}{l_1^*(\gamma_2 - \gamma_1) + 2\gamma_2} \tag{17}$$

and it is easy to demonstrate that $\lambda_2^* > \lambda_1^*$ is equivalent to

$$l_2^* > l_2 \tag{18}$$

For a load of l_2, the delay at network 2 becomes (Eq. (10))

$$T_2 = \frac{l_2 + 2}{2\gamma_2} n_{h_2} \tag{19}$$

The total system delay is the sum delays in the two networks

$$T_e^* = T_1^* + T_2$$

Using the value of T_2 from Eq. (19) and substituting the value of l_2 from Eq. (17), after some algebra we get

$$T_e^* = \frac{l_1^* + 2}{2\gamma_1} \left[1 + \frac{2}{l_1^*(\frac{\gamma_2}{\gamma_1} - 1) + 2\frac{\gamma_2}{\gamma_1}} \frac{n_{h_2}}{n_{h_1}} \right] n_{h_1} \tag{20}$$

Comparing Eq. (16) with (6), and (20) with (7), we have: if $\lambda_2^* > \lambda_1^*$ then

$$[l_1^*, n_{h_1}, \gamma_1] -o- [l_2^*, n_{h_2}, \gamma_2] \equiv [l_1^*, n_{h_e}, \gamma_1]$$

where

$$n_{h_e} = \left[1 + \frac{2}{l_1^*(\frac{\gamma_2}{\gamma_1} - 1) + 2\frac{\gamma_2}{\gamma_1}} \frac{n_{h_2}}{n_{h_1}} \right] n_{h_1}$$

and the equivalent window size will be

$$w_e = \left[1 + \frac{2}{l_1^*(\frac{\gamma_2}{\gamma_1} - 1) + 2\frac{\gamma_2}{\gamma_1}} \frac{n_{h_2}}{n_{h_1}} \right] w_1$$

Case III: $\lambda_2^* < \lambda_1^*$

In this case network 2 is the bottleneck for the throughput. The analysis is similar to case II; indices 1 and 2 should simply be replaced with each other.

In general, when several networks are connected together in series, the bottleneck for the throughput is the network with the smallest traffic handling capacity, and we have

$$\sum_{i=1}^{N}(-o-)\ [l_i^*, n_{h_i}, \gamma_i] \equiv [l_e^*, n_{h_e}, \gamma_e]$$

Let i^* be such that

$$\lambda_{i^*}^* = \min_{1 \leq i \leq N} \{\lambda_i^*\} \tag{21}$$

and define α_{i^*} to be

$$\alpha_{i^*} = \sum_{i=1}^{N} \left[\frac{2}{l_{i^*}^*(\frac{\gamma_i}{\gamma_{i^*}} - 1) + 2\frac{\gamma_i}{\gamma_{i^*}}} \frac{n_{h_i}}{n_{h_{i^*}}} \right] \tag{22}$$

then $l_e^* = l_{i^*}^*$; $n_{h_e} = \alpha_{i^*} \cdot n_{h_{i^*}}$; $\gamma_e = \gamma_{i^*}$; and the equivalent window size, the throughput and the delay will be, respectively, $w_e = \alpha_{i^*} \cdot w_{i^*}$; $\lambda_e^* = \lambda_{i^*}^*$; and

$$T_e^* = \frac{l_{i^*}^* + 2}{2\gamma_{i^*}} \alpha_{i^*} \cdot n_{h_{i^*}}.$$

In the special case when $\gamma_i = \gamma$, $i=1, \ldots, N$, the network with the lowest throughput handling capacity is the same one with the lowest load factor, i.e.

$$l_{i^*}^* = \min_{1 \leq i \leq N} \{l_i^*\} \tag{23}$$

where i^* is defined in Eq. (21). That is, the network with the lowest (maximum) load factor is the bottleneck. It can be shown that in this case the actual load factors of all the networks are equal to $l_{i^*}^*$, the maximum load factor of the bottleneck net, and we have [5]

$$l_i = l_{i^*}^* < l_i^* \qquad i=1, 2, \ldots, N \tag{24}$$

Furthermore, the equivalent network has a path length equal to the sum of the path lengths of the component nets, and the equivalent transmission rate is $\gamma_e = \gamma$. The throughput and the delay of the equivalent network for this special case are shown in Table 1.

4. Parallel Connections of Networks

As in the analysis of series connections, we assume that the operation of each of the parallel-connected networks is independent of the others. Parallel connections is similar to the case where a user has a number of networks (or a number of different, independent paths on the same network) at his disposal to send his messages to the receiver. Note that we assume the networks do not share common windows.

Unless the maximum load factors of the networks are equal, the analysis becomes complicated and no simple expression for the equivalent network has been derived; therefore, in what follows we will assume $l_i^* = l^*$ for all i.

	SERIES —o—	PARALLEL —≡—
SPECIAL CASE	$\gamma_i = \gamma_j = \gamma \quad 1 \leq i, j \leq N$ $\sum_{i=1}^{N} (-o-) [l_i^*, n_{h_i}, \gamma] => [l_e^*, n_{h_e}, \gamma_e]$ Let i^* be such that $\lambda_{i^*}^* = \min_i \{\lambda_i^*\}$ $(l_{i^*}^* = \min_i \{l_i^*\})$ $\gamma_e = \gamma$ $l_e^* = l_{i^*}^*$ $n_{h_e} = \sum n_{h_i}$ $w_e = \frac{\sum n_{h_i}}{n_{h_{i^*}}} w_{i^*}$ $\lambda_e^* = \lambda_{i^*}^*$ $T_e^* = \frac{l_{i^*} + 2}{2\gamma} \sum n_{h_i}$	$l_i^* = l_j^* = l^* \quad 1 \leq i, j \leq N$ $\sum_{i=1}^{N} (-\equiv-) [l_i^*, n_{h_i}, \gamma_i] => [l_e^*, n_{h_e}, \gamma_e]$ $\gamma_e = \sum \gamma_i$ $l_e^* = l^*$ $n_{h_e} = \sum n_{h_i}$ $w_e = \sum w_i$ $\lambda_e^* = \frac{l^*}{l^* + 2} \sum \gamma_i$ $T_e^* = \frac{l^* + 2}{2\sum \gamma_i} \sum n_{h_i}$
GENERAL CASE	$\sum_{i=1}^{N} (-o-) [l_i^*, n_{h_i}, \gamma_i] => [l_e^*, n_{h_e}, \gamma_e]$ Let i^* be such that $\lambda_{i^*}^* = \min_i \{\lambda_i^*\}$ and $\alpha_{i^*} = \sum_{i=1}^{N} \frac{2}{l_i^*(\frac{\gamma_i}{\gamma_{i^*}} - 1) + 2\frac{\gamma_i}{\gamma_{i^*}}} \frac{n_{h_i}}{n_{h_{i^*}}}$ $\gamma_e = \gamma_{i^*}$ $l_e^* = l_{i^*}^*$ $n_{h_e} = \alpha_{i^*} \cdot n_{h_{i^*}}$ $w_e = \alpha_{i^*} \cdot w_{i^*}$ $\lambda_e^* = \lambda_{i^*}^*$ $T_e^* = \frac{l_{i^*} + 2}{2\gamma} \alpha_{i^*} \cdot n_{h_{i^*}}$	

Table 1– A Network Algebra ($\gamma_i > 0$ and $w_i > 0$).

Consider N networks connected together in parallel. When each network carries its maximum traffic, the total traffic carried by the system becomes the sum of the traffic handling capacities of each network, hence we have

$$\lambda_e^* = \sum_{i=1}^{N} \lambda_i^* = \sum_{i=1}^{N} \frac{l_i^*}{l_i^* + 2} \gamma_i$$

Because $l_i^* = l^*$, $1 \leq i \leq N$, we have

$$\lambda_e^* = \frac{l^*}{l^* + 2} \sum_{i=1}^{N} \gamma_i \tag{25}$$

The system delay can be calculated by considering the fact that a fraction of λ_i^*/λ_e^* of the total traffic λ_e^* that is sent through network i experiences a delay of T_i^*, hence the average system delay becomes

$$T_e^* = \sum_{i=1}^{N} \frac{\lambda_i^*}{\lambda_e^*} T_i^*$$

Using the values of λ_i^* and T_i^* from Eqs. (6) and (7), and considering the fact that $l_i^* = l^*$ for all i, we have

$$T_e^* = \frac{l^* + 2}{2\sum_i \gamma_i} \sum_i n_{h_i} \tag{26}$$

Eqs. (25) and (26) indicate that for the equivalent network the channel transmission rate is the sum of the transmission rates of the networks, and the equivalent number of hops is also the sum of the number of hops in each network.

Thus we have: if $l_i^* = l^*$, $i = 1, \ldots, N$, then

$$\sum_{i=1}^{N} (-\equiv-) [l^*, n_{h_i}, \gamma_i] \equiv [l^*, \sum_i n_{h_i}, \sum_i \gamma_i]$$

The equivalent window size will be

$$w_e = \frac{\sum_i n_{h_i}}{n_{h_j}} w_j \quad \text{for any } j$$

or

$$w_e = \sum_i w_i$$

The traffic handling capacity and the delay of the equivalent network can be found by using Eqs. (25) and (26).

Table 1 shows a summary of our results for the series and/or parallel connections of networks when $w_i > 0$ and $\gamma_i > 0$. For the series connections the network with the lowest maximum traffic handling capacity is the bottleneck of the system. In the special case that the channel transmission rates of all networks are identical, the number of hops of the equivalent network becomes the same as the sum of the number of hops of the networks. For the parallel connections we have shown only the results for the special case that the maximum load factors of all the networks are identical; the results for the general case do not allow a simple equivalence to be found.

5. Conclusion

In this paper we characterized systems composed of series and/or parallel connections of computer networks and developed a network algebra as an aid in the systematic study of internetting. For series connections of networks we found an isolated equivalent network with the same behavior. It was observed that the throughput of this system is determined by a bottleneck net, the network with the smallest traffic handling capacity. Eqs. (18) and (24) show that the load factor of each of the connected nets is less than their maximum load factor (i.e., $l_i \leqslant l_i^*$). Considering the fact that the load factor is equal to the average number of messages in the nodes of a net (Section 2), we have

$$n_i = n_{h_i} l_i \leqslant n_{h_i} l_i^* = w_i \qquad (27)$$

(n_i is the average number of messages in network i). Therefore, the average number of message is less than (or equal to, in the bottleneck net) the window size. Because the window size w_i determines the amount of buffer storage used in network i, Eq. (27) shows that part of the resources of the network are wasted.

The analysis of networks connected in parallel was carried out only for the special case when the maximum load factors of the connected networks are identical. We showed that a system composed of parallel connections of such networks is identical to a single network with a path length and transmission rate equal to sum of the path lengths and sum of the transmission rates of its components, and its maximum load factor is equal to the maximum load factor of each component network.

Clearly, any connection of series/parallel networks can be reduced to a single equivalent network, using the methods given in this paper.

Appendix

In this appendix we analyze a cyclic queue model of the window mechanism described in Section 2.

We model the communication path between a source S and destination D by n_h tandem queues. For each message that arrives at the destination node, an ACK is generated and piggybacked on the messages that are being transmitted from the destination node D to the source node S. Similar to Section 2, we assume that traffic between nodes S and D is symmetric and we model the reverse path from node D to node S as another set of tandem queues. Because the total number of unacknowledged messages is restricted to the window size w, under heavy traffic conditions there are exactly w messages on the paths, of which, on the average, $w/2$ are being transmitted to D and the other $w/2$ are carrying ACKs to node S. This model is shown in Fig. A.1, where each channel (hop) is represented by a single server queue with service rate γ. Assuming the service time is exponentially distributed, the model can easily be solved for the statistics of interest [8]. In particular, the probability that any one of the queues is empty can be found to be

$$P_0 = \frac{2n_h - 1}{w + 2n_h - 1}$$

The throughput of each queue is therefore $(1-P_0)\gamma$. This quantity is also equal to the throughput of the network under heavy traffic conditions. Using the notation of Section 2, we have

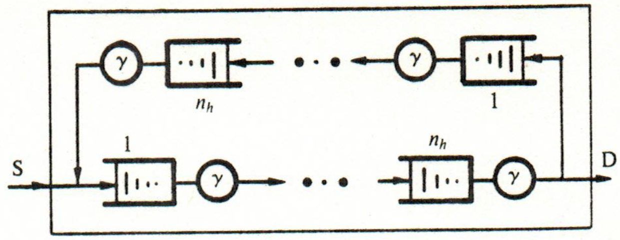


Figure A.1- A Closed Network of Queues Model for the Window Mechanism

$$\lambda^* = (1 - \frac{2n_h - 1}{w + 2n_h - 1})\,\gamma = \frac{w}{w + 2n_h - 1}\,\gamma$$

By using the Little's Result [9], we can easily find the (maximum) network delay T^* to be

$$T^* = \frac{w + 2n_h - 1}{2\gamma}$$

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