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ON THE CAPACITY OF ONE-HOP ALOHA PACKET RADIO NETWORKS WITH ADJUSTABLE TRANSMISSION POWER *

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ABSTRACT

In this paper we investigate two possible policies for realizing an arbitrary traffic matrix in a Slotted ALOHA broadcast packet radio network: full connectivity and limited transmission power. The performance for the fully connected (point-to-point) network is the same as the known result for a centralized network and allows a maximum throughput of $1/e$. The other approach, wherein we give each node sufficient power to just reach his destination, allows a maximum throughput proportional to the logarithm of the number of (active) nodes in the network. These results, which are derived analytically, are then verified by simulation, showing excellent agreement.

1. Introduction

One of the major problems in the effective utilization of computer resources is the distribution of those resources to the user. This problem has been greatly alleviated by the advent of communication networks but communication among these users in a local environment still remains a problem. The concept of broadcast packet radio for local access was first utilized in the ALOHA system [ABRA 70] and more recently, the Advanced Research Projects Agency of the Department of Defense has undertaken a project to investigate the use of more general broadcast packet radio systems [KAHN 77, KAHN 78]. A packet radio network consists of many packet radio units (PRUs)** sharing a common radio channel such that when one unit transmits, many other units will hear the packet, even though it is addressed to only one of them. This feature, inherent in broadcast systems, in conjunction with the fact that channel access control is neither automatic nor free, results in destructive interference when several packets are received simultaneously.

Many studies have been conducted to evaluate the capacity (maximum achievable throughput) of one-hop centralized communication networks using broadcast radio as the communication medium. In [ABRA 70] the capacity of fully-connected one-hop centralized pure ALOHA was found to be $1/2e$ and in [ROBE 75] the corresponding result for Slotted ALOHA networks was found to be $1/e$. In [LAM 74] we find an extensive analysis for the fully connected one-hop centralized slotted ALOHA access scheme and in [TOBA 74, KLEI 75] we find similar results for Carrier Sense Multiple Access (CSMA). Local access networks usually have centralized traffic requirements (the central node often being a gateway to the main network).

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** All components of the packet radio network (i.e., stations, repeaters and terminals) use a common device for channel access. This device is called the Packet Radio Unit.

In [GITM 75, TOBA 78b] we find some capacity results for two-hop Slotted ALOHA centralized nets and these results are extended to CSMA in [TOBA 78a].

On the other hand, some applications of broadcast radio nets, (such as ship-to-ship communication or remote sensing networks) have arbitrary point-to-point traffic requirements. This kind of broadcast network has received little attention in the literature. In [KLEI 78] we derive the capacity for a *random* network (i.e., the nodes are randomly located in space, giving rise to random connectivity) using multi-hop Slotted ALOHA and find that we can achieve throughput proportional to the square root of the number of nodes in the network by restricting the transmission range. In fact, we find that the optimum number of nodes to have in range (i.e., the nodal degree) is about six. In [SILV 79b] we derive similar results for *regular* networks (i.e, nodes are regularly located and connected to a fixed number of neighbors), again finding a square root behavior and also that the nodal degrees should be minimized (subject to a connectivity constraint). The performance of any network will depend on the traffic requirements and in [SILV 79a] we derive bounds on the capacity of a single hop point-to-point network, for an 'optimal' traffic matrix, showing that we can achieve a throughput proportional to the number of nodes in the network.

In this paper we consider point-to-point networks, satisfying *arbitrary* traffic matrices, in which the nodes are sufficiently close that they can communicate in one hop. We investigate whether the nodes should transmit at full power or restrict their power to just allow communication to their intended destination. By restricting the range we expect higher performance since the interference will be reduced. This is the phenomemon that we have observed in other studies [KLEI 78, SILV 79a, SILV 79b].

2. The Network Model

The networks that we study in this paper have a random topology, which may be thought of as either representatives of the set of all possible networks or as snapshots of a mobile network. In order to model the requirement that the network should be able to handle an arbitrary traffic pattern, we assume a uniform traffic matrix. Our traffic model is, then, of the (instantaneous) communication requirement between some active subset of the total number of nodes in the network (non-active nodes are ignored).

A network is a set of n (active) nodes (with n even to allow pairing) randomly located in a unit hypersphere. These nodes are then randomly paired to represent communicating pairs of nodes. Having generated the network and the traffic matrix, we satisfy the communication requirement by suitable choice of transmission power. We consider two approaches that will allow this paired communication to be reallized: i) give every node sufficient power to be able to reach *every* other node in the network; or ii) give each node sufficient power to just reach his communication partner.

Once the network is established we have one additional parameter to specify - the probability that a node will transmit in any slot. (This corresponds to the offered channel traffic, randomized so that Slotted ALOHA will operate correctly and resolve previous conflicts caused by to simultaneous transmissions.) In order to compute the throughput we use the 'heavy traffic model', which corresponds to assuming that all (active) nodes are always busy, but which transmit in any given slot depending on this transmission probability. We denote the transmission probability for node i as p_i.

Nodal Throughput: Consider an arbitrary node (say node i) in the network. The probability that this node correctly receives a packet from his partner (say node j) in any slot, is given by:

$$s_i = Pr\{j \text{ transmits}\}\ Pr\{i \text{ does not transmit}\}\ Pr\{\text{none of } i\text{'s neighbors transmits}\}$$

$$= p_j\ (1-p_i) \prod_{k \in N_i} (1-p_k) \tag{1}$$

where N_i is the set of nodes that i can hear (excluding his partner j). We are assuming here that a node either hears a transmission or hears nothing. (In a real network the reception process is not discrete but depends on noise levels etc.)

For the heavy traffic model, s_i corresponds to the (received) throughput γ_i for this node. Thus the total network throughput, γ, is given by:

$$\gamma = \sum_{i=1}^{n} \gamma_i = \sum_{i=1}^{n} s_i \tag{2}$$

3. Completely Connected Topologies

One approach to satisfying an arbitrary random traffic matrix is to give every node sufficient transmission power so that all the nodes in the network hear when any one transmits. This corresponds to the model of [ROBE 75, ABRA 77] and the total network throughput will therefore be $1/e$. We proceed to show that our approach gives the same result.

Since the environment for each node is identical, we assume $p_i = p$. The number of nodes that can interfere with a given transmission is $n-2$, so the throughput for each node is:

$$\gamma_i = p\ (1-p)\ (1-p)^{n-2} \tag{3}$$

In order to set the offered traffic in any environment to be the optimum value of one packet per slot [ABRA 70, LAM 74, YEMI 78], we use a transmission probability of $p = 1/n$. We then have:

$$\gamma_i = \frac{1}{n} \left(1 - \frac{1}{n} \right)^{n-1} \tag{4}$$

As the throughput for each node is identically distributed, the network throughput, γ ,is simply:

$$\gamma = \left(1 - \frac{1}{n} \right)^{n-1} \tag{5}$$

which has the expected asymptotic behavior of $1/e$ for large networks.

4. Limited Transmitter Power

Another approach for arbitrary traffic matrices is to limit the power of each transmitter so that it exactly reaches its destination (we are assuming that reception is a two-state process, either you can or cannot hear a transmission). In Figure 1 we show a two-dimensional network of 10 nodes generated in this manner by the simulation program described in section 5; the lines joining pairs of nodes represent the traffic matrix and hence the transmission radii.

Since the networks we consider are homogeneous, the throughput for all nodes is identically distributed; we, therefore, drop the subscripts corresponding to the particular node under investigation. We set the offered traffic in any environment to unity, by selecting the transmission probability to be $1/k$ for a node that interferes with k others when he transmits, including himself and his transmission partner. In [SILV 79b] we studied other transmission policies and found that the scheme used here had the highest throughputs. Using the notation $\gamma(k)$ to represent the throughput for a node which interferes with k nodes and making the assumption that both nodes of a partnership hit the same number of nodes*, we obtain the following expression for the throughput:

$$\gamma(k) = I \frac{1}{k} \left(1 - \frac{1}{k}\right) \tag{6}$$

where I, the interference factor, is the interference contribution from nodes other than the node itself. We can think of this factor as background interference. If we assume that the interference encountered at any node is independent of the degree of that node, then the expected thoughput for any node in the network, γ_{node}, is given by:

$$\gamma_{node} = I \sum_{k=2}^{n} h_k \, \gamma(k) \tag{7}$$

where h_k is the probability that a node hits (interferes with) k other nodes when he transmits (note that he always hits himself and his partner).

We now proceed to find this hitting distribution. Consider an arbitrary node, P, in the network and rank the $n-1$ other nodes in order of their distance from P. If P is paired with his kth neighbor, he will interfere with (hit) exactly $k+1$ nodes when he transmits. As P is equally likely to be paired with any of the nodes, the hitting distribution is given by:

$$h_k = \frac{1}{n-1} \qquad k=2,3,\ldots,n \tag{8}$$

We can now compute the throughput.

$$\gamma_{node} = I \sum_{k=2}^{n} h_k \frac{1}{k} \left(1 - \frac{1}{k}\right)$$

$$= \frac{I}{n-1} \left[\sum_{k=2}^{n} \frac{1}{k} \left(1 - \frac{1}{k}\right)\right] \tag{9}$$

In [SILV 79b] we show that the interference factor, I, for networks where the interference heard by a node is independent of the degree of that node and the transmission probability is $1/k$, is given by:

$$I = \prod_{k=3}^{n} \left(1 - \frac{k-2}{n-2} \frac{h_k}{k}\right)^{n-2} \tag{10}$$

Thus the nodal throughput is given by:

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* As both nodes are transmitting at the same range, certainly the *expected* number hit by a transmission will be the same.

$$\gamma_{node} = \frac{1}{n-1}\left[\prod_{k=3}^{n}\left(1-\frac{k-2}{n-2}\,\frac{1}{k}\,\frac{1}{n-1}\right)^{n-2}\right]\left[\sum_{k=2}^{n}\frac{1}{k}\left(1-\frac{1}{k}\right)\right] \tag{11}$$

Since the throughput for each node is identically distributed, the total network throughput, γ, will be given by $n\gamma_{node}$.

$$\gamma = \frac{n}{n-1}\left[\prod_{k=3}^{n}\left(1-\frac{k-2}{n-2}\,\frac{1}{k}\,\frac{1}{n-1}\right)^{n-2}\right]\left[\sum_{k=2}^{n}\frac{1}{k}\left(1-\frac{1}{k}\right)\right] \tag{12}$$

With some manipulation, we find that the asymptotic behavior for large networks is given by:

$$\lim_{n\to\infty}\gamma = \frac{\log(n) + C - \frac{\pi^2}{6}}{e} \tag{13}$$

where C is Euler's constant. This can be approximated by:

$$\gamma \approx \frac{\log(n) - 1}{e} \tag{14}$$

The above results were derived with no reference to the dimensionality of the network. We can therefore achieve a throughput *logarithmically* proportional to the network size for all networks satisfying an arbitrary traffic pattern by exact adjustment of transmission range. It must be pointed out, however, that the throughput for all pairs of nodes in the network is not the same. Nodes that are close together (and thus have high transmission probabilities since they do not interfere with many other nodes) will achieve higher throughputs than those that are far apart (recall that the background interference is uniform for all nodes in the network). Even the node with the smallest throughput (in the worst case this node will hit $n-2$ other nodes) will have a throughput of:

$$\gamma(n-2) = \frac{1}{ne} \tag{15}$$

for large networks, which is the same as that for the fully connected case (in which *every* node achieves a throughput of $1/ne$). Thus the node experiencing the worst performance will be doing no worse than for the fully connected case, whereas nodes close together will far exceed this throughput. We are currently investigating the capacity of the network if some 'fairness' requirement is imposed.

5. Simulation

In order to check the validity of this model, we developed a simulation program to compute the throughputs for these networks. This program operates as follows (described for a two-dimensional network).

A randomly generated network is located inside the unit circle and pairs of nodes are then randomly matched. With this pairing, the transmission radii are determined so that communication can take place, and the adjacency matrix is computed. We then determine the transmission probabilities, based on the number of nodes within range of the node. From this we compute the success probabilities for each node and hence the network throughput. This process is repeated many times and the throughput, averaged over several networks, is computed.

In Figure 2 we have plotted throughput against network size, showing analytical and simulation results for a two-dimensional network averaged over 50 networks. Notice the excellent agreement between the model and simulation.

6. Conclusions

We have investigated two possible policies for supporting an arbitrary traffic matrix in a point-to-point broadcast packet radio network: full connectivity and limited transmission power. The performance for the fully connected network is the same as the known result for a centralized network and allows a maximum throughput of $1/e$. The other approach, wherein we give each node power sufficient to just reach his destination, allows a maximum throughput proportional to the logarithm of the number of (active) nodes in the network. This behavior was determined by use of an analytical model which was then verified by simulation.

One problem with this approach is that the network throughput is not allocated 'fairly' in that those nodes which are close together (and thus with higher transmission probabilities), will achieve higher throughputs than those which are more distant. In fact, it can be shown that we cannot increase the throughput of these high interference nodes without reducing the overall performance.

The traffic matrix considered in this paper is, in some sense, a 'worst case' as it shows no locality. (In fact the worst case would be if every node wished to talk to a node at the other extreme of the network, forcing the fully connected case.) If, as would probably be the case in reality, the traffic matrix exhibited some degree of locality, the capacity would exceed that shown here. In [SILV 79a] we investigated the case of extreme locality and attempted to find the 'best' possible traffic matrix. We were able to bound the performance from above and below by linear functions of the number of nodes in the network and found a traffic matrix that achieved a capacity between these bounds.

In a *real* one-hop point-to-point packet broadcast network we would therefore expect that we could obtain a capacity which grows somewhere between logarithmically and linearly with respect to the number of nodes in the network, depending on the traffic characteristics.

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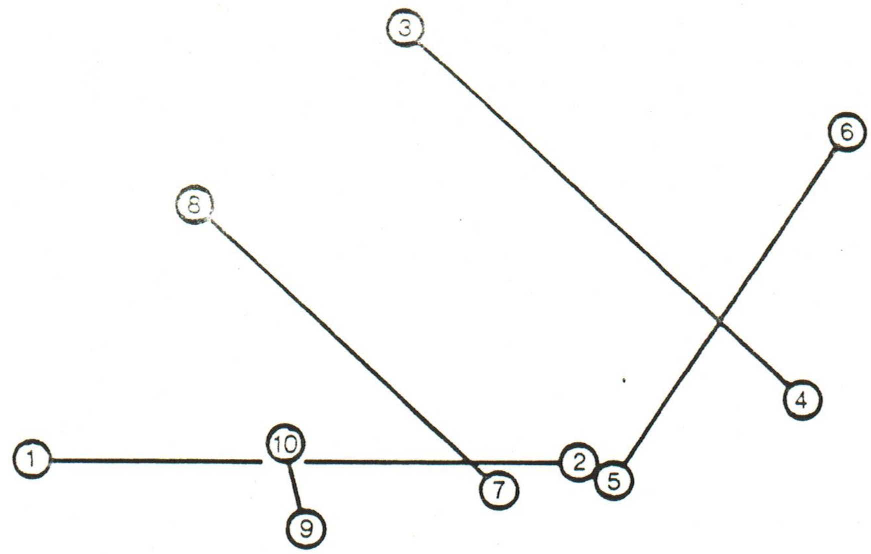

Figure 1. 10 Node Network Showing (Adjusted) Transmission Radii

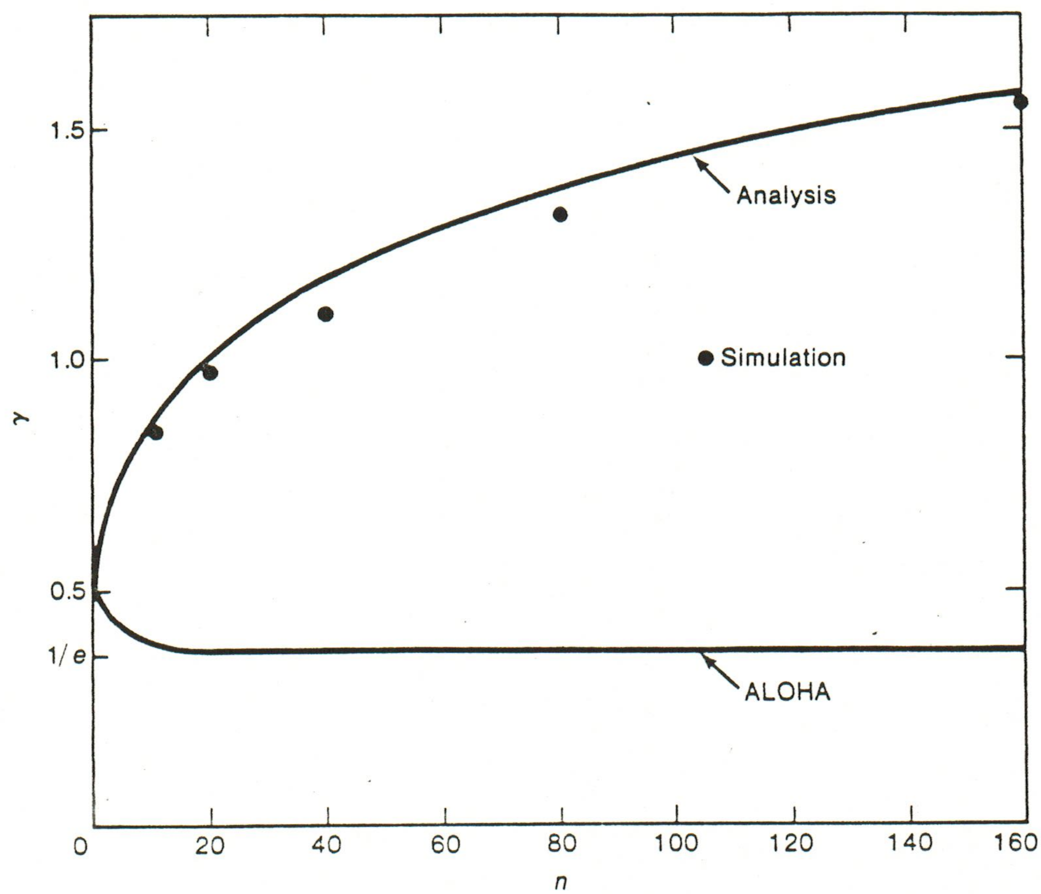

Figure 2. Analytical and Simulation Results for 2-dimensional Networks