Journal of High Speed Networks 1 (1992), 61–77
IOS Press

PERFORMANCE ANALYSIS OF SINGLE-HOP WAVELENGTH DIVISION MULTIPLE ACCESS NETWORKS*

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Abstract

Wavelength division multiple access (WDMA) provides a way to tap the huge bandwidth of an optical fiber by simultaneously operating on multiple channels at different wavelengths, with each channel running at the speed of the electronics of an end user station. This paper presents a mathematical model which approximates WDMA networks with general hardware configurations and arbitrary traffic patterns. Packets which cannot be transmitted upon arrival are blocked (i.e., lost) immediately. We first study the case of a uniform traffic matrix and observe that, when the number of wavelengths is fewer than the number of stations, it is better to have both tunable transmitters and tunable receivers, rather than having only either one of them tunable. Furthermore, we find that only a small number of tunable transmitters and receivers per station is needed to produce performance close to the upper bound. We then construct a general traffic model and propose an iterative solution procedure. A case of hot-spot traffic is studied using this model. We find that adding more resources to the hot-spot node will help improve its performance, but only to a limited extent determined by the traffic imbalance. The match between the model and simulation results are shown to be excellent.

Keywords: Wavelength Division Multiplexing, Performance Analysis, Fiber Optics

1. Introduction

The rapid development of lightwave technology offers the potential of a huge amount of bandwidth in a single optical fiber. It is conceivable that we could construct multiple access networks with a total capacity of around 50 terabits per

* This work was supported by the Defense Advanced Research Projects Agency under Contract MDA 903-87-C0663, Parallel Systems Laboratory.

second by using the low-loss passband of optical fibers (1200–1600 nm) [1]. An obstacle to realizing such high-capacity networks lies in the bottleneck at the electronic interface, which can modulate/demodulate the light at a mere fraction of the optical bandwidth. Therefore, to tap the bandwidth potential of optical fibers, the network architecture must employ some form of concurrency, i.e., the ability to simultaneously convey a multitude of distinguishable messages. One such approach, called Wavelength Division Multiple Access (WDMA), could achieve this by operating on multiple channels at different wavelengths, with each channel running at the speed of the electronics of an end user station. By assembling a large number of wavelength-multiplexed channels, WDMA carries the potential of providing the network capacity required by future applications.

One class of WDMA networks is the multi-hop network [2, 3], which is constructed by setting the transmitters and/or receivers of a station to be tuned at certain fixed wavelengths. A link is formed between two nodes when a transmitter of one node and a receiver of the other node both tune to the same wavelength. The way these transmitters and receivers are tuned defines an interconnection pattern. A packet may be routed through several intermediate nodes before it is delivered to its destination. An early proposal for the interconnection pattern consisited of several stages connected through a Perfect Shuffle [4]. However, there is no a *priori* reason to be restricted to this interconnection pattern. Two other papers [5, 6] propose schemes to optimize the logical connectivity by (slowly) retuning the transmitters and receivers of the stations adaptively to the traffic. Another class of WDMA networks [7–10] assumes single-hop communications which employs tunable transmitters and/or receivers with rapid tuning to dynamically set up connections between stations on a per packet basis. Both the single-hop and multi-hop networks can achieve an aggregate throughput substantially larger than the electronic speed of a single station. An advantage of single-hop over multi-hop communications is that multi-hop implies longer routes and thus larger propagation delays, which is the dominating delay component in high-speed networks. In this paper we consider only single-hop cases.

Ramaswami and Pankaj [11] compared having either tunable transmitters only, or tunable receivers only, or both, assuming each station is equipped with only one transmitter and one receiver. Chlamtac and Ganz [12] discussed the design alternatives of WDMA star networks where each station can have multiple transmitters and receivers and some finite buffers. Both of these two previous studies were conducted only for the case of a uniform traffic matrix. The purpose of this paper is to present a mathematical model for WDMA networks to examine the effects of resource contention (of transmitters, receivers, and wavelengths) under general traffic patterns. Each station may have its own hardware configuration and traffic requirement. Our model ignores any specific media access protocol by assuming that each station has perfect knowledge of the current status of all the resources in the system. This assumption is reasonably good for the case of a packet switch where the physical distance is small and stations can learn the status of the resources from information broadcast by a centralized controller. The model serves as an upper bound on performance when the system is a network which covers a larger geographical area.

The rest of the paper is organized as follows. In Section 2, we describe the system configuration and assumptions to be used in the mathematical model. Section 3 presents the analysis of networks with stations having multiple transmitters and receivers for the uniform traffic case. A general model is constructed in Section 4 and an iterative procedure is proposed to solve it for the general traffic case. In Section 5, a hot-spot traffic case is then studied using the general model. Section 6 gives the conclusions.

2. The System Model and the Solution Method

The system considered here consists of N stations attached to a broadcast medium (fiber bus or star coupler). The number of wavelengths is equal to W. Node[1] i has t_i transmitters and r_i receivers, each of which may be tunable to any wavelength or which may be tuned to a single fixed wavelength. We assume that a stream of packets arrive to node i following a Poisson process with rate λ_i packets per unit time. The packet length is exponentially distributed with mean $1/\mu$, the same for all nodes. We shall choose the average packet length as the time unit of the system by setting $\mu = 1$ throughout the analysis. A packet arriving at node i is addressed to destination node j with probability x_{ij}, $1 \le i, j \le N$. Define

$$\phi_i \triangleq \sum_{j=1}^{N} \lambda_j x_{ji} \qquad 1 \le i \le N$$

as the intensity of generated traffic that is destined for node i. For a packet to be transmitted and successfully received, the three following conditions must all be satisfied simultaneously: (i) there is a free wavelength in the system, (ii) there is a free transmitter, at the source node, which can access that free wavelength, and (iii) there is a free receiver, at the destination node, which can also access that same free wavelength. We assume there is no buffering at each node. Upon a packet's arrival, it is transmitted immediately if all the three conditions above are true (remember that we have assumed a "perfect" access scheme); otherwise the packet is blocked (i.e., lost) immediately. We assume that each station has complete knowledge of the status (busy or idle) of all the wavelengths, transmitters, and receivers in the system. The throughput of the system, which is defined as the average number of successful packets transmitted per unit time, will be used as the performance measure to compare systems with different configurations and different traffic patterns.

Let the random variable K be the number of busy wavelengths in the system in steady state. Let $p_k \triangleq Prob[K = k]$, $0 \le k \le W$. Knowing the number of busy wavelengths does not completely describe the state of the system since we also need the current status of the transmitters and receivers of each node. However, we will make the approximation that K is a Markov chain. In this analysis, we will also approximate many of the transition rates of this chain and then provide an exact solution under these approximations. Given that the system is in state k, $0 \le k \le W-1$, and given a specific free wavelength, we define $\alpha_k^{(i)}$ as the

[1] The words *node* and *station* will be used interchangeably throughout this paper.

probability that an arriving packet at node i finds at least one of its transmitters free which can access that free wavelength, and $\beta_k^{(j)}$ as the probability that a packet destined for node j arriving at a source node finds, upon its arrival, a free receiver at node j which can access that same free wavelength. We recognize that these two probabilities should properly be computed as a joint probability; we choose to approximate them by assuming independence of the underlying events. Let σ_k denote the transition rate from state k to state $k+1$ due to the transmission of a new packet. We first note that $\lambda_i x_{ij}$ is the rate of new packets generated by node i and addressed to node j. The probability that this new packet is successfully transmitted is approximately equal to $\alpha_k^{(i)}\beta_k^{(j)}$. Therefore, under the assumption that all the free wavelengths are equally favored for the transmission of a new packet, σ_k can be calculated as follows:

$$\sigma_k = \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i x_{ij} \alpha_k^{(i)} \beta_k^{(j)} \qquad 0 \le k \le W \tag{1}$$

We see that the evolution of K forms a Markov chain which is a birth-death process whose state transition diagram is shown in Fig. 1. Solving this birth-death process [13], we have

$$p_k = p_0 \prod_{i=0}^{k-1} \frac{\sigma_i}{(i+1)\mu} \tag{2}$$

where

$$p_0 = \left[1 + \sum_{k=1}^{W} \prod_{i=0}^{k-1} \frac{\sigma_i}{(i+1)\mu}\right]^{-1} \tag{3}$$

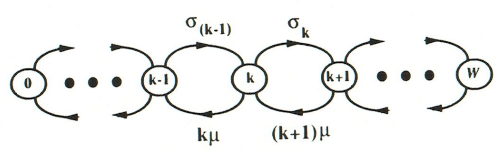

Fig. 1. State transition diagram for number of busy wavelengths in the system

The throughput of the system, S which is also equal to the average number of busy wavelengths in the system, can be calculated by

$$S = \sum_{k=0}^{W} k p_k \tag{4}$$

This, then, is the general setup for our solution. It remains to find σ_k and hence S. This we do in the next two sections.

3. The Uniform Traffic Case

In this section we study the uniform traffic case where packets arrive to a station following a Poisson process with rate λ packets per unit time (the same for all stations). A packet will travel from its source station to any of the N stations (including the source itself) with equal probability, i.e., $x_{ij} = 1/N$, $1 \le i,j \le N$. (Setting $x_{ij} = 1/(N-1)$, $1 \le i,j \le N$, $i \ne j$ does not change the results below.)

3.1. *Tunable Transmitters and Receivers*

Here we consider the case where each node is equipped with q ($q \le W$) tunable transmitters and q tunable receivers, each of which can tune to any of the W wavelengths. The $\alpha_k^{(i)}$'s and $\beta_k^{(i)}$'s are now the same for all stations by symmetry, which we denote by α_k and β_k, respectively. To get the α_k and β_k, we first note, given that the system is in state k, that it is implied that there are also k transmitters and k receivers currently busy in the system. When $k < q$, α_k (β_k) is equal to one because there must be always a free transmitter (receiver) at any node. For the cases $k \ge q$, since there is a total of Nq transmitters (receivers) in the system, we know that the probability that any single transmitter (receiver) is busy equals k/Nq. Therefore, the probability that all the transmitters (receivers) of a given node are busy is approximately equal to $(k/Nq)^q$. One minus this gives us α_k (β_k). Thus, we have the following approximation:

$$\alpha_k = \beta_k = \begin{cases} 1 & 0 \le k \le q-1 \\ 1 - (\frac{k}{Nq})^q & q \le k \le W-1 \end{cases} \tag{5}$$

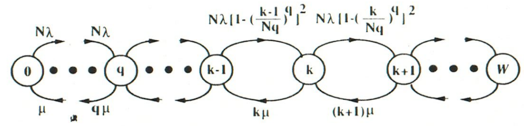

Fig. 2. State transition diagram for tunable transmitters and receivers

The transition rates σ_k can be calculated using (1) and (5), and the corresponding state transition diagram is shown in Fig. 2. Solving this Markov chain, we get

$$p_k = p_0 \frac{(N\rho)^k}{k!} \qquad 0 \le k \le q$$

$$p_k = p_0 \frac{(N\rho)^k}{k!} \prod_{i=q}^{k-1} \left[1 - \left(\frac{i}{Nq}\right)^q\right]^2 \qquad q+1 \le k \le W$$

where $\rho = \lambda/\mu$ and

$$p_0 = \left[\sum_{k=0}^{q} \frac{(N\rho)^k}{k!} + \sum_{k=q+1}^{W} \frac{(N\rho)^k}{k!} \prod_{i=q}^{k-1} \left(1 - \left(\frac{i}{Nq}\right)^q\right)^2\right]^{-1}$$

The throughput S can be calculated from (4).

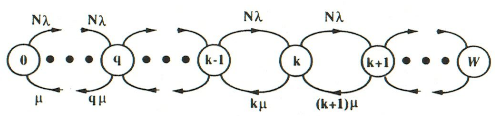

Fig. 3. An upper bound, the W-server loss system

An *achievable* upper bound on the throughput can be obtained by assuming all nodes have W tunable transmitters and receivers. In this case, $\alpha_k = \beta_k = 1$, which corresponds to a W-server loss system [13] where each wavelength corresponds to a server. Figure 3 shows the corresponding state transition diagram. Solving this, we have

$$p_k = p_0 \frac{(N\rho)^k}{k!} \qquad 0 \le k \le W$$

where

$$p_0 = \left[\sum_{k=0}^{W} \frac{(N\rho)^k}{k!} \right]^{-1}$$

The blocking probability of this upper bound system equals

$$p_W = \frac{(N\rho)^W / W!}{\sum_{k=0}^{W} (N\rho)^k / k!}$$

which is the well-known *Erlang B formula* [13].

In Fig. 4 and 5 we plot the throughput versus the total offered load for $N = 50$, $W = 10$ and $N = 50$, $W = 50$, respectively. We show the ideal upper bound on throughput as equal to the input load up to the point where the load equals the total system bandwidth; beyond that point, any additional traffic is clearly lost. We can see that a small q (much smaller than W) is enough to produce a result close to the achievable upper bound where $q = W$. This is because, in the uniform traffic case, the probability that more than a few packets are going to the same destination at the same time is very small, and only a small number of transmitters and receivers are required at each node.

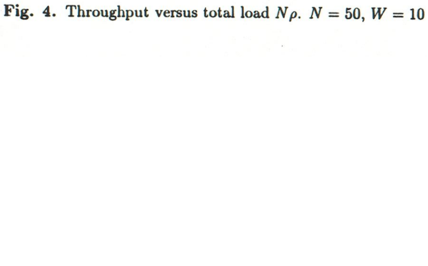

Fig. 4. Throughput versus total load $N\rho$. $N = 50$, $W = 10$

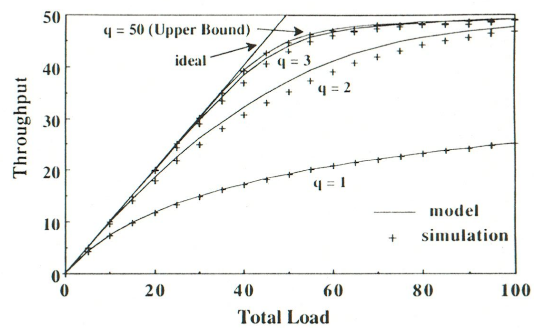

Fig. 5. Throughput versus total load $N\rho$. $N = 50$, $W = 50$

3.2. *Tunability on One Side Only*

In this section we consider the same uniform traffic case except that each station now has only tunable transmitters or receivers, but not both. We begin with the case where each node is equipped with one tunable transmitter and f ($f \le W$) fixed tuned receivers. Each receiver in a station is tuned to a different fixed wavelength and the receivers in the whole system are tuned in a uniform way such that the number of recievers tuned to each wavelength is the same, which equals Nf/W (assumed to be an integer).

By the same arguments as in the previous subsection, α_k can be easily (but approximately) derived from (5) by setting $q = 1$.

$$\alpha_k = 1 - \frac{k}{N} \qquad 0 \le k \le W - 1$$

To get β_k requires a bit of different reasoning. For $k < f$, β_k equals one because the total number of busy receivers in the sytem is fewer than the number of receivers each station has. To transmit a new packet, the source node can just tune its transmitter to the free wavelength of any idle receiver at the destination. For the case $k \ge f$, recall that all the receivers are tuned in a uniform way over all the wavelengths; therefore, we know that, given that the system is in state k (i.e., there are currently k busy wavelengths), the probability that the fixed wavelength at an arbitrary receiver at the destination is busy equals k/W. The probability that wavelengths at the receivers of a given node are all busy is approximately $(k/W)^f$, and one minus this gives us β_k as follows:

$$\beta_k = \begin{cases} 1 & 0 \le k \le f-1 \\ 1-(\frac{k}{W})^f & f \le k \le W-1 \end{cases}$$

By switching the roles of transmitters and receivers in the discussion above, we can easily obtain the α_k and β_k for the case of multiple fixed transmitters and one tunable receiver per node, which are equal to the β_k and α_k listed above, respectively; the two systems are "duals" of each other. Therefore, the state transition diagrams of those two cases are exactly the same and is shown in Fig. 6. Solving this Markov chain under our approximation, we have

$$p_k = p_0 \frac{(N\rho)^k}{k!} \prod_{i=0}^{k-1} \left(1-\frac{i}{N}\right) \qquad 1 \le k \le f$$

$$p_k = p_0 \frac{(N\rho)^k}{k!} \left[\prod_{i=0}^{k-1} \left(1-\frac{i}{N}\right)\right] \left[\prod_{i=f}^{k-1} \left(1-\left(\frac{i}{W}\right)^f\right)\right] \qquad f+1 \le k \le W$$

where $\rho = \lambda/\mu$, and

$$p_0 = \left[1 + \sum_{k=1}^{f} \frac{(N\rho)^k}{k!} \prod_{i=0}^{k-1} \left(1-\frac{i}{N}\right) + \sum_{k=f+1}^{W} \frac{(N\rho)^k}{k!} \left[\prod_{i=0}^{k-1} \left(1-\frac{i}{N}\right)\right] \left[\prod_{i=f}^{k-1} \left(1-\left(\frac{i}{W}\right)^f\right)\right]\right]^{-1}$$

The throughput can be calculated from (4).

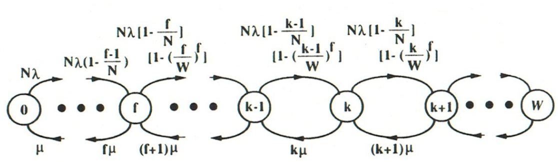

Fig. 6. State transition diagram for tunability on one side only

Figure 7 shows the case in which the number of wavelengths is small ($W = 10$) compared to the number of nodes ($N = 50$) in the system. We see that there is an interval in the light load range where multiple fixed receivers is better than one tunable receiver because not many wavelengths are in use and a station with

multiple receivers can receive more than one packet at a time. However, as the load increases the average number of wavelengths in use increases too, and it is better to have a tunable receiver than multiple fixed receivers because the wavelengths those fixed receivers are tuned to may be all in use (by other stations) and a given station could not receive any packet even though not all of its receivers were busy. Figure 8 shows the case where $N = 50$ and $W = 25$ on a different scale. Once again we see the importance of going to a single tunable receiver at heavy load. When the number of wavelengths becomes the same as the number of nodes in the

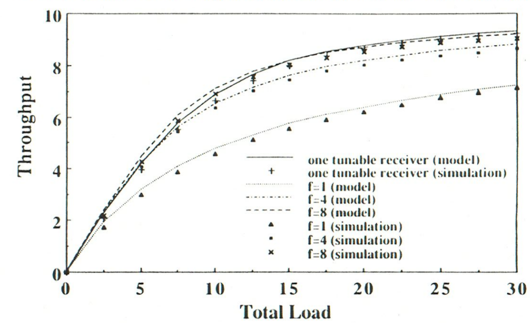

Fig. 7. Throughput versus total load $N\rho$. $N = 50$, $W = 10$, and one tunable transmitter

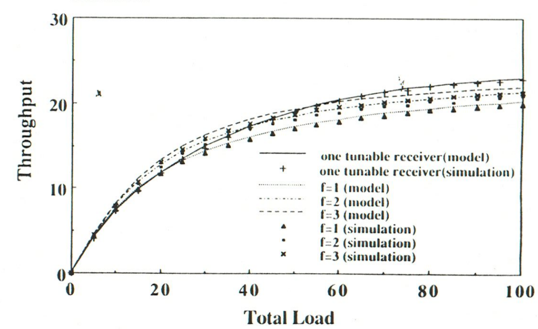

Fig. 8. Throughput versus total load $N\rho$. $N = 50$, $W = 25$, and one tunable transmitter

system ($W = N = 50$) as plotted in Fig. 9, wavelength is no longer the scarce resource and the performance of having tunability on both sides is the same as on one side only. In this case having multiple fixed receivers is always better. Note the excellent match between the results from our approximations and simulations in the figures.

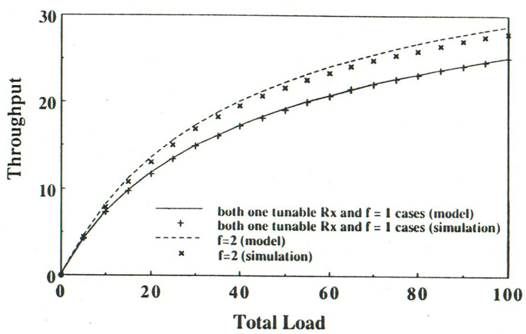

Fig. 9. Throughput versus total load $N\rho$. $N = 50$, $W = 50$, and one tunable transmitter

4. The General Traffic Case

4.1. *The Model*

Here we consider the general traffic case. We assume that node i has t_i tunable transmitters and r_i tunable receivers only. Let λ_i^* and ϕ_i^* denote the number of packets successfully transmitted and received by node i per unit time, respectively. Clearly, $S = \sum_{i=1}^{N} \lambda_i^* = \sum_{i=1}^{N} \phi_i^*$. Thus $\alpha_k^{(i)}$ can be approximated as follows:

$$\alpha_k^{(i)} = \begin{cases} 1 & 0 \le k \le t_i - 1 \\ 1 - (k\lambda_i^*/t_i S)^{t_i} & t_i \le k \le \min(t_i S/\lambda_i^*, W-1) \\ 0 & \min(t_i S/\lambda_i^*, W-1) \le k \le W-1 \end{cases} \tag{6}$$

The quantity $(k\lambda_i^*/S)$ is the average number of busy transmitters of node i, given that the system is in state k. The $(k\lambda_i^*/t_i S)$ equals the probability that any single transmitter of node i is busy given that the system is in state k. Therefore, $(k\lambda_i^*/t_i S)^{t_i}$ is approximately equal to the probability that all of node i's transmitters are busy, and one minus that gives us the $\alpha_k^{(i)}$. For those k's where the value $(k\lambda_i^*/t_i S)$ is greater than one, we set the $\alpha_k^{(i)}$ to zero. The $\beta_k^{(i)}$'s can be derived in a similar way:

$$\beta_k^{(i)} = \begin{cases} 1 & 0 \le k \le r_i - 1 \\ 1 - (k\phi_i^*/r_i S)^{r_i} & r_i \le k \le \min(r_i S/\phi_i^*, W-1) \\ 0 & \min(r_i S/\phi_i^*, W-1) \le k \le W-1 \end{cases} \tag{7}$$

Note that when the traffic is uniform, $\lambda_i^*/S = \phi_i^*/S = 1/N$, $1 \le i \le N$ by symmetry, and (6) and (7) both reduce to (5).

We now derive λ_i^*. Let $U^{(i)}$ denote the number of busy transmitters of node i in steady state with probability mass function (pmf) $u_m^{(i)} \triangleq Prob[U^{(i)} = m]$. We will approximate $U^{(i)}$ as a Markov process. Define

$$p_{k|m} \triangleq Prob[K = k \mid K \ge m] = p_k / \sum_{j=m}^{W} p_j$$

Let $\eta_m^{(i)}$ be the transition rate for $U^{(i)}$ from state m to state $m+1$. $\eta_m^{(i)}$ can be calculated as follows:

$$\eta_m^{(i)} = \lambda_i \sum_{j=1}^{N} x_{ij} \sum_{k=m}^{W-1} \beta_k^{(j)} p_{k|m}$$

The transition rate from state m to $m-1$ is just $m\mu$, the aggregrate rate at which any busy transmitter of node i will finish its transmission first. Figure 10 shows the state transition diagram. Solving this, we have

$$u_m^{(i)} = u_0^{(i)} \prod_{n=0}^{m-1} \frac{\eta_n^{(i)}}{(n+1)\mu}$$

where

$$u_0^{(i)} = \left[1 + \sum_{m=1}^{t_i} \prod_{n=0}^{m-1} \frac{\eta_n^{(i)}}{(n+1)\mu} \right]^{-1}$$

λ_i^* can be obtained from

$$\lambda_i^* = \sum_{m=0}^{t_i} m u_m^{(i)} \tag{8}$$

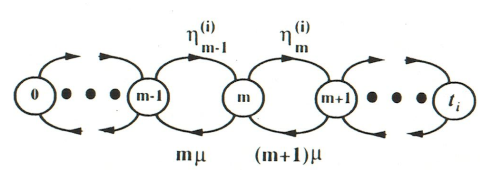

Fig. 10. State transition diagram for $U^{(i)}$

The ϕ_i^* can be derived in almost the same way. Let $V^{(i)}$ denote the number of busy receivers of node i in steady state with pmf $v_m^{(i)} \triangleq Prob[V^{(i)} = m]$. Define $\tau_m^{(i)}$ as the transition rate for $V^{(i)}$ from state m to state $m+1$. The $\tau_m^{(i)}$ can be calculated as follows:

$$\tau_m^{(i)} = \sum_{j=1}^{N} \lambda_j x_{ji} \sum_{k=m}^{W-1} \alpha_k^{(j)} p_{k|m}$$

The transition rate from state m to $m-1$ is just $m\mu$, the aggregrate rate at which any busy receiver of node i will finish its reception first. Figure 11 shows the state transition diagram. Solving this, we have

$$v_m^{(i)} = v_0^{(i)} \prod_{n=0}^{m-1} \frac{\tau_n^{(i)}}{(n+1)\mu}$$

where

$$v_0^{(i)} = \left[1 + \sum_{m=1}^{r_i} \prod_{n=0}^{m-1} \frac{\tau_n^{(i)}}{(n+1)\mu}\right]^{-1}$$

The ϕ_i^* can be obtained from

$$\phi_i^* = \sum_{m=0}^{r_i} m v_m^{(i)} \tag{9}$$

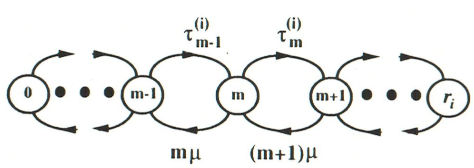

Fig. 11. State transition diagram for $V^{(i)}$

However, we do not really have the p_k's in the first place to compute those λ_i^* and ϕ_i^* because they depend on each other. In the next subsection, we propose an iterative procedure to solve for these steady state probabilities.

4.2. *An Iterative Procedure*

We define $p_k(n)$, $\lambda_i^*(n)$, $\phi_i^*(n)$, $\alpha_k^{(i)}(n)$, and $\beta_k^{(i)}(n)$ as the values obtained for these quantities at the end of the nth iteration. We start with some initial estimates $p_k(0)$, $\lambda_i^*(0)$, and $\phi_i^*(0)$. One simple initial estimate is to set $p_k(0) = 1/(W+1)$, $0 \le k \le W$, $\lambda_i^*(0) = \lambda_i$, and $\phi_i^*(0) = \phi_i$, $1 \le i \le N$. The iterative procedure is as follows:

1. Let $n = 1$.
2. Construct $\alpha_k^{(i)}(n)$ and $\beta_k^{(i)}(n)$ from $\lambda_i^*(n-1)$ and $\phi_i^*(n-1)$ using (6) and (7). Solve for $p_k(n)$ from (1), (2), and (3).
3. With $p_k(n)$, $\alpha_k^{(i)}(n)$, and $\beta_k^{(i)}(n)$, solve the Markov chains in Fig. 10 and 11 to get $\lambda_i^*(n)$ and $\phi_i^*(n)$.
4. If the difference between $p_k(n)$, $\lambda_i^*(n)$, $\phi_i^*(n)$, and $p_k(n-1)$, $\lambda_i^*(n-1)$, $\phi_i^*(n-1)$, respectively, are less than pre-specified thresholds, then stop. Otherwise, set $n = n+1$ and go to step 2.

We do not have proof of the convergence of the procedure above. However, for all the experiments presented in the next section, this procedure converges all the time, and the solutions are very close to the simulation results.

5. The Hot-Spot Traffic Case

Here we use the general model just described to study the special case of a "hot-spot" traffic pattern where a large portion of traffic is addressed to a specific node called the hot-spot node. The other $N-1$ nodes are called "plain" nodes. Without loss of generality, let node 1 be the hot-spot node. We assume all $\lambda_i = \lambda$, $1 \le i \le N$. From the generated traffic from all the nodes, a fraction of b is assumed to go to the hot-spot node, and the rest goes to the other nodes uniformly, i.e., $x_{i1} = b$, $x_{ij} = (1-b)/(N-1)$, $1 \le i \le N$, $2 \le j \le N$. Each node has one tunable transmitter and one tunable receiver except node 1, which may have more than one tunable receiver. That is, $t_i = 1$, $1 \le i \le N$, $r_1 \ge 1$, and $r_j = 1$, $2 \le j \le N$. The effect of various values of b and r_1 on the system performance is investigated below.

Figure 12 shows the relationship between the throughput and total load for the case of $N = 50$, $W = 10$, and $r_1 = 1$. We can see that as the bias b gets larger, the total throughput of the system is degraded. This is because, while the single receiver of the hot-spot node is overloaded, there is not enough traffic generated for exchange among the other nodes.

Since the receiver of the hot-spot node is now the scarce resource, we next study the effect of increasing the number of receivers at the hot-spot node. In Fig. 13 and 14 we plot the *received* throughput (i.e., ϕ_1^*) of the hot-spot node (node 1 in our example) versus the total load for the cases of $N = 50$, $W = 10$, $b = 0.2$, and $b = 0.8$, respectively. We note that, by increasing the number of receivers at the hot-spot node, its throughput can be improved. However, as the load increases, we see that the received throughput of the hot-spot node saturates at some value no matter how large a number of receivers it has. This is because when the total load is very heavy, the throughput of the system approaches W, and the received throughput of each node saturates at some value determined by the traffic imbalance. Putting in a lot more receivers at the hot-spot node will not help further increase its received throughput. To compute this saturated throughput for the hot-spot node (node 1), we let the load λ go to infinity. Define H as the number of busy receivers of node 1 in steady state with pmf $\pi_m \triangleq Prob[H = m]$, $0 \le m \le r_1$. The transition rate of H moving out of state m can be calculated as follows: We first note that as λ goes to infinity, there are W packets in transmission in the system all the time. Given $H = m$, we know that there are m packets going

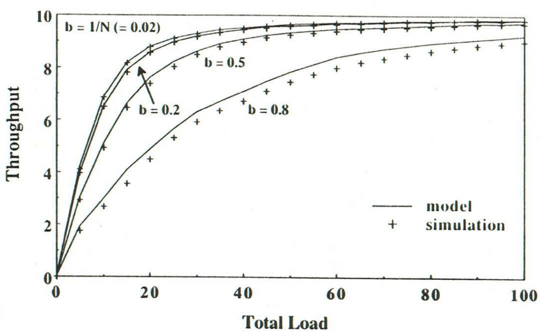

Fig. 12. Throughput versus the total load $N\rho$. $N = 50$, $W = 10$, $r_1 = 1$

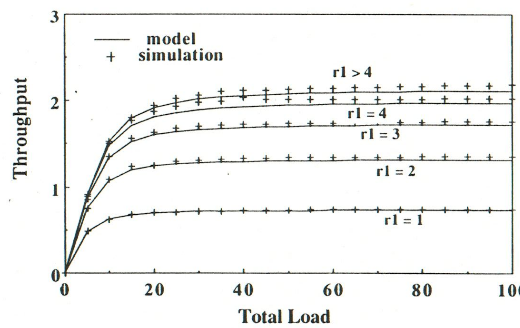

Fig. 13. Throughput of the hot-spot node versus total load $N\rho$. $N = 50$, $W = 10$, $b = 0.2$

to node 1 and $W - m$ to the others. The number of busy receivers of node 1 will increase by one when the transmission of any of the $W - m$ packets addressed to the plain nodes is finished first (with rate $(W - m)\mu$) and the wavelength just freed is immediately grabbed by a new packet addressed to node 1 (packets arrive infinitely fast since $\lambda \to \infty$), the probability of which we denote by y_m. To compute y_m, we note that, right after the transmission of any of the $W - m$ packets addressed to the plain nodes is finished, there are currently $(N-1)-(W-m)+1 = (N - W + m)$

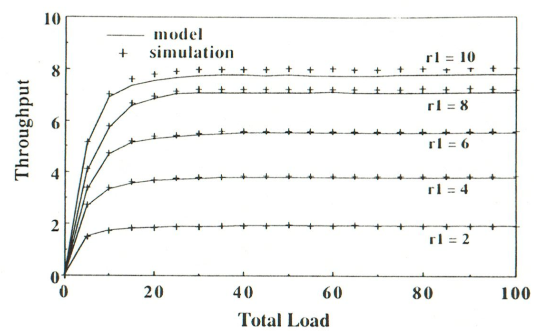

Fig. 14. Throughput of the hot-spot node versus total load $N\rho$. $N = 50$, $W = 10$, $b = 0.8$

plain nodes whose receivers are idle. The probability that the next arriving packet is addressed to the hot-spot node is equal to

$$y_m = \frac{b}{b + (N - W + m)\frac{1-b}{N-1}}$$

Therefore, the rate of H moving from m to $m+1$ equals $(W - m)\mu y_m$. On the other hand, H will decrease by one if the transmission of any of the m packets to node 1 finishes first (with rate $m\mu$) and the free wavelength is then occupied by a new packet addressed to a plain node, the probability of which is just $(1 - y_{m-1})$ because there are $(N-1)-(W-m) = (N - W + m - 1)$ plain nodes whose receivers are idle. Thus, the rate of H moving from m to $m-1$ is $m\mu(1 - y_{m-1})$. The state transition diagram is shown in Fig. 15. Solving this, we have

$$\pi_m = \pi_0 \binom{W}{m} \prod_{j=0}^{m-1} \frac{y_j}{1 - y_j} \qquad 1 \le m \le r_1$$

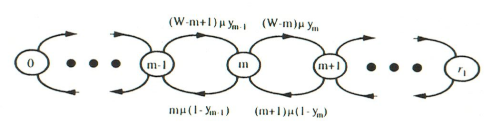

Fig. 15. State transition diagram for H

where

$$\pi_0 = \left[1 + \sum_{m=1}^{r_1} \binom{W}{m} \prod_{j=0}^{m-1} \frac{y_j}{1-y_j}\right]^{-1}$$

The real received throughput of node 1 as $\lambda \to \infty$, S_1, can be calculated from

$$S_1 = \sum_{m=0}^{r_1} m\pi_m$$

Figure 16 shows S_1 versus the number of receivers of node 1 for the case of $N = 50$ and $W = 10$. We see that, given an extremely heavy load and a large number of receivers, the hot-spot node can achieve a larger asymptotic throughput as the fraction of traffic addressed to the hot-spot node gets larger.

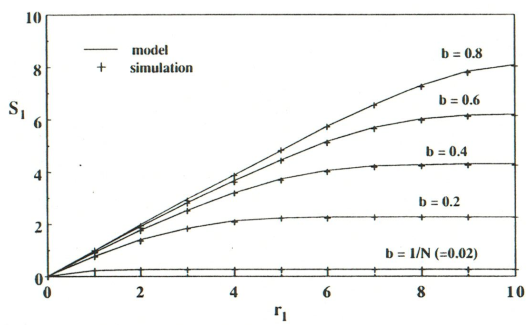

Fig. 16. The relationship between the limiting value S_1 and r_1. $N = 50$, $W = 10$

6. Conclusions

Optical fiber provides a huge amount of potential bandwidth and the bottleneck to tapping this enormous bandwidth lies at the electronic interface of the end stations. WDMA holds great promise for achieving large-scale concurrency in an optical fiber by allowing multiple communication pairs to exchange data on different channels simultaneously. In this paper we first built a model to analyze the uniform traffic case. We found that it is better to have both tunable transmitters and tunable receivers than having only one or the other tunable when the number of wavelengths is smaller than the number of nodes (which is most likely the case in the near future [14]). Also a small number of tunable transmitters and receivers at each station is enough to produce performance close to the upper bound. We then constructed a model for systems with general hardware configurations and

arbitrary traffic patterns. An iterative procedure was proposed to solve the model numerically. We used this model to study a special hot-spot traffic case. We saw that traffic imbalance could degrade the performance of the system. Adding more receivers to the hot-spot node helps improve its performance, but only to a limited extent determined by the traffic imbalance. The match between the results from our approximations and simulations was shown to be excellent.

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