Packet Switching in Radio Channels: Part IV—Stability Considerations and Dynamic Control in Carrier Sense Multiple Access

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Abstract—In two companion papers a method for multiplexing a population of terminals communicating with a central station over a packet-switched radio channel was introduced; this method is known as Carrier Sense Multiple Access (CSMA). CSMA, as with ALOHA multiaccess broadcast channels, has the unfortunate property that the throughput falls to zero as the channel load increases beyond a critical value. The dynamic behavior and stability of slotted ALOHA channels have been studied extensively and have led to a definition of stability. In this paper, similar techniques are used to analyze CSMA, which is shown to have a behavior not unlike that of ALOHA. However, contrary to ALOHA channels where steady-state performance is badly degraded when true stability is to be guaranteed, hence requiring dynamic control, we find that CSMA provides excellent stable performance even with as large a population as 1000 terminals. Furthermore, we study a simple adaptive retransmission control procedure which provides a significantly improved channel performance which is insensitive to the population size.

1. INTRODUCTION

IN PART I of this series [1, 2], a packet-switching technique referred to as Carrier Sense Multiple Access (CSMA) was introduced and studied in detail. This technique enables efficient sharing of a data communication channel by a large population of bursty users in a ground radio environment; this environment is characterized by a propagation delay between terminals which is very small compared to the transmission time of the packet. Briefly, CSMA reduces the level of interference (caused by overlapping packets) in the random multiaccess environment by allowing terminals to sense the carrier due to other users' transmissions; based on this channel state information (busy or idle), the terminal takes an action prescribed by the particular CSMA protocol being used. In particular, a terminal never transmits when it senses that the channel is busy. In Part I we described and analyzed two protocols referred to as nonpersistent and p-persistent CSMA; the performance of these was given in terms of channel capacity and throughput-delay tradeoffs. The analysis was based on a number of model assumptions. First, we assumed that our traffic source consists of an infinite number of users who collectively form an independent Poisson source. This is merely an approximation to the case of a large but finite population in which each user generates packets infrequently and each packet can be successfully transmitted in a time interval much less than the average time between successive packets generated by that user. Each user in the infinite population was thus assumed to have at most one packet requiring transmission at any time (including any previously blocked packet). Secondly, we assumed that *steady state* conditions prevail. With this inifinite population model we could then determine *analytically* the capacity of the channel under the various protocols; we also established a measure of the delay performance in terms of the average number of transmissions and schedulings incurred by a packet until success. The analysis of packet delay, however, proved to be complex and simulation techniques were used. The results derived from simulation were also based on the assumption that whenever the system reaches a stationary state in which it remains for a reasonable length of time (namely, the simulation run time of 10,000 packet transmission time, determined empirically), then the system must have reached steady state; the throughput-delay results so derived then represented the *equilibrium* channel performance. We realized that many of these assumptions (infinite population, Poisson input, stationarity, stability) merely represent approximations to the physical situation, and that without them the mathematical analysis becomes untractable and solutions are difficult to come by. The question to be asked at this point is, in view of our assumptions, how valid are our results?

Random multiaccess broadcast channels are characterized by the fact that the throughput goes to zero for large values of channel load. This is due to a positive feedback of traffic which is inherent to the operation of these systems. If, for some reason, the rate of retransmission of packets increases, interference may become so frequent that fewer transmissions are successful and yet more users shift to the retransmission mode, thus further increasing the retransmission rate, etc. Moreover, extensive simulation runs performed on slotted ALOHA channels with an infinite population [3] have shown that the assumption of channel equilibrium may not always be valid. In fact, after some finite time period of *quasi-stationarity* conditions, the channel will drift into saturation with probability one

Thus, as realized by Kleinrock and Lam [4], the (assumed) equilibrium throughput-delay results are not sufficient to characterize the performance of the infinite population. A more representative measure of channel performance is the stability-throughput-delay tradeoff. In their paper, Kleinrock and Lam defined a mathematical model which characterizes the slotted ALOHA channel state by a single variable. (The model is similar to the linear feedback model studied by Metcalfe who gave a steady-state analysis of the slotted ALOHA

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system behavior [5].) Based on the model, a theory was proposed in [4] which characterizes the instability phenomenon by defining stable and unstable channels; in *stable* channels, the equilibrium throughput-delay results (i.e., obtained under the channel equilibrium assumption) are achievable over an infinite time horizon, while in *unstable* channels, such channel performance is achievable only for some finite time period before the channel goes into saturation (in which it remains for a long period of time); for unstable channels, a stability measure is defined and a computational procedure for its calculation is given. Using this stability measure, the stability-throughput-delay tradeoff for an unstabilized channel is obtained.

Independently using the same model, Carleial and Hellman [6] analyzed the behavior of slotted and unslotted ALOHA channels; their results reconfirm the bistable nature of unstable channels as defined above. In their recent paper, Fayolle *et al.* [7] give yet another theoretical treatment of the instability of slotted ALOHA channels with infinite populations and give some control policies to recover stability.

The purpose of this paper is to apply the stability theory defined in [4] in order to predict the behavior of CSMA channels, discuss the conditions under which we can guarantee stability and finally give the performance of these guaranteed stable channels. (We restrict ourselves to the nonpersistent CSMA protocol because of its simplicity in analysis and implementation, as well as its relatively high efficiency.) For this we first review, in Section 2, the nonpersistent protocol and describe the mathematical model. In Section 3, we focus on the analysis of nonpersistent CSMA which allows us to obtain *analytically*, for the finite population model, the throughput-delay performance as well as the effect of the retransmission delay and of the population size on the performance. Next, in Section 4, we apply the stability theory introduced in [3] and [4] to CSMA, discuss the channel's behavior, and give an analysis for the channel performance during the uptime of unstable channels; this finally provides us with the actual stability-throughput-delay tradeoff. Furthermore, in Section 5, we study an adaptive retransmission control procedure which stabilizes the channel and provides us with a significantly improved channel performance, shown to be practically independent of the population size.

2. THE MODEL

Briefly, the idea in nonpersistent CSMA [1] is to limit repeated interference among the packets by always rescheduling (into the future) a packet which finds the channel busy. Thus a ready terminal (one with a packet to be transmitted) senses the channel and proceeds as follows.

1) If the channel is sensed idle, it transmits the packet.
2) If the channel is sensed busy, then the terminal schedules the retransmission of the packet to some later time, and then repeats the algorithm.

A slotted version of the nonpersistent CSMA is considered here in which the time axis is (mini-) slotted and the slot size is τ seconds (the propagation delay). Packets, assumed to be of fixed length, require a transmission time of T slots. (Typically, T is on the order of 100; this corresponds then to a ratio of propagation delay τ to transmission time T of 0.01 [1].) All terminals are synchronized and are forced to start transmission only at the beginning of a slot. When a packet's arrival occurs during a slot, the terminal senses the channel at its arrival and then operates according to the protocol described above. Without loss of generality, we assume that the sensing operation is instantaneous on this relatively wide-band channel.

We consider a user population consisting of M users (terminals), all in line of sight and within range of each other. Each such user can be in one of two states: *backlogged* or *thinking*. In the thinking state, a user generates## a new packet in a slot with probability σ. A user whose packet *either* had a channel collision *or* was blocked because of a busy channel is said to be backlogged. A backlogged user remains in that state until he successfully transmits the packet at which time he switches to the thinking state. Thus, a user in the backlogged state cannot generate a *new* packet for transmission. The rescheduling delay of a backlogged packet is assumed to be geometrically distributed, i.e., each backlogged user is scheduled to resense the channel in the current slot with a probability ν; as specified in the description of the protocol, a retransmission would result only if the channel is sensed idle. The memoryless property of the geometrically distributed retransmission delay will permit a simple state description for the mathematical model as can be seen in the sequel. We shall further assume that a terminal learns about its success or failure instantaneously at the end of its transmission period; i.e., no time-out period for receiving acknowledgement packets is assumed. However, the terminal stays in the backlogged state during the transmission period.

Let N^t be a random variable called the *channel backlog* representing the number of backlogged users at the beginning of slot t. The channel input rate at time t, defined as the average number of new packets generated by the thinking users at time t, is denoted by S^t. Obviously, S^t decreases as N^t increases. For the purpose of this study, we shall assume M, σ and ν to be time invariant.

In slotted ALOHA, the action that a terminal takes pertaining to the transmission of a packet (either newly generated or rescheduled) is completely independent of the *state of the channel** (busy or idle). Therefore, in slotted ALOHA, the channel backlog over a (*large*) slot (equal to the transmission time of a packet) is a Markov process with *homogeneous stationary* transition probabilities and serves as the state description for the system. In CSMA, on the contrary, the action taken by a terminal depends on the state of the channel. For example, assume that some new packets are generated by users in the thinking state during a transmission period. Those

##At the time a user in the thinking state generates a new packet, it senses the channel, and if the latter is idle, it transmits the packet with probability one.

*The *state of the channel* (busy, idle) is to be distinguished from the *state of a user* (thinking, backlogged) or the *state of the system* defined as the channel backlog.

users sensing the channel busy are blocked and switch to the backlogged state with probability one. Thus we note that the transition probabilities of the process representing the channel backlog are not independent of the state of the channel. We derive these transition probabilities below. The discrete *state space* of the system consists of the integers $\{0, 1, 2, \cdots, M\}$.

3. ANALYSIS

Consider the "imbedded" slots defined to be the first slot of each idle period (see Fig. 1). The intervals of time between two consecutive imbedded slots are defined as cycles [1]. These cycles, of course, are of random length. Consider one such cycle and let t_e denote the first slot. N^{t_e} denotes the state of the system at t_e. Let I denote the length of the idle period (in slots). The length of the cycle is then $I + T + 1$. (This last slot, represented in Fig. 1 by dashed lines, accounts for the propagation delay; it is only one slot after the end of transmission that the channel will be sensed idle.) By definition, no terminal is ready during the interval $[t_e, t_e + I - 2]$; however, at least one terminal becomes ready in the last slot of the idle period, i.e., at time $t_e + I - 1$. (Vertical arrows in Fig. 1 represent arrivals.) For all $t \in [t_e, t_e + I - 1]$ we have $N^t = N^{t_e}$. All terminals which become ready at $t_e + I - 1$ will sense the channel idle and will transmit at the beginning of slot $t_e + I$. Given that $N^{t_e+I-1} = i$, the probability that some terminal is ready is given by

$$\Pr\{\text{some terminal is ready}/N^{t_e+I-1} = i\}$$
$$= 1 - (1-\nu)^i(1-\sigma)^{M-i}. \tag{1}$$

Conditioned on the fact that a transmission starts at slot $t_e + I$, let $R = (r_{ik})$ be the one-step transition matrix between slot $t_e + I - 1$ and $t_e + I$. That is

$$r_{ik} \triangleq \Pr\{N^{t_e+I} = k/N^{t_e+I-1} = i\}. \tag{2}$$

For $i = 0, 1, 2, \cdots$, we have

$$r_{ik} = \begin{cases} 0, & k < i \\ \dfrac{(1-\sigma)^{M-i}[1-(1-\nu)^i]}{1-(1-\nu)^i(1-\sigma)^{M-i}}, & k = i \\ \dfrac{\binom{M-i}{k-i}(1-\sigma)^{M-k}\sigma^{k-i}}{1-(1-\nu)^i(1-\sigma)^{M-i}}, & k > i. \end{cases} \tag{3}$$

On the other hand, all ready terminals in the interval $[t_e + I, t_e + I + T]$ will sense the channel busy and will be blocked from transmitting;† these terminals remain in the backlogged state if they were already backlogged, or switch to the backlogged state if they were in the thinking state. For any $t \in [t_e + I + 1, t_e + I + T]$, let $Q = (q_{ik})$ be the one-step transition matrix defined by

†According to the slotted nonpersistent CSMA described above, packets arriving during slot $t_e + I + T$ will sense the channel busy and will be blocked. Consequently, the idle period between two transmission periods will always be at least one slot.

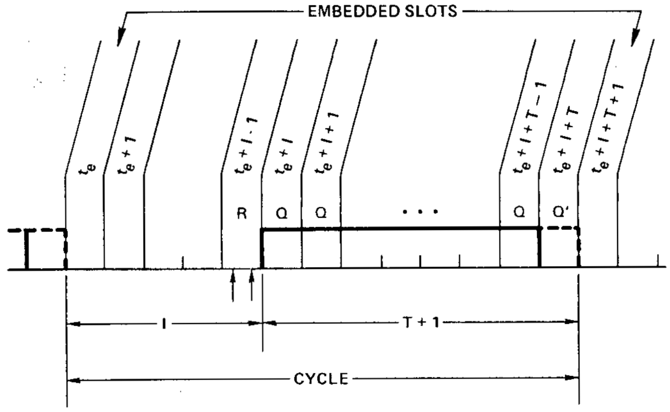

Fig. 1. The Imbedded Markov Chain in Slotted Nonpersistent CSMA.

$$q_{ik} \triangleq \Pr\{N^t = k/N^{t-1} = i\}. \tag{4}$$

For $i = 1, 2, \cdots$, under the convention that $\binom{M}{k} = 0$ for $k > M$, we have

$$q_{ik} = \begin{cases} 0, & k < i \\ \binom{M-i}{k-i}(1-\sigma)^{M-k}\sigma^{k-i}, & k \geq i. \end{cases} \tag{5}$$

Finally, let $Q' = (q_{ik}')$ be the one-step transition matrix corresponding to the last slot of the cycle

$$q_{ik}' \triangleq \Pr\{N^{t_e+I+T+1} = k/N^{t_e+I+T} = i\}. \tag{6}$$

Since this step corresponds to the last slot of the transmission period, two types of events exist: (i) if there exists a single transmitting terminal, that transmission is successful and the corresponding terminal switches to the thinking state; (ii) terminals that are ready during the last slot will still sense the channel busy at that slot and will become backlogged if they were not already in that state. The probability of success over the transmission period is dependent on the state of the system at the time the transmission begins, i.e., on N^{t_e+I-1} which equals N^{t_e}. Conditioned on $N^{t_e} = n$, the probability of success, denoted by $P_s(n)$ is given by

$$P_s(n) = \frac{(1-\nu)^n(M-n)\sigma(1-\sigma)^{M-n-1} + n\nu(1-\nu)^{n-1}(1-\sigma)^{M-n}}{1-(1-\nu)^n(1-\sigma)^{M-n}}. \tag{7}$$

The last step transition probabilities therefore also depend on

N^{t_e} and are expressed for $j = 1, 2, \cdots, M$ as

$$q_{jk}'(n) = \begin{cases} 0, & k < j-1 \\ (1-\sigma)^{M-j} P_s(n), & k = j-1 \\ (M-j)\sigma(1-\sigma)^{M-j-1} P_s(n) \\ \quad + (1-\sigma)^{M-j}[1-P_s(n)], & k = j \\ \binom{M-j}{k-j} \sigma^{k-j}(1-\sigma)^{M-k}[1-P_s(n)] \\ \quad + \binom{M-j}{k-j+1} \sigma^{k-j+1}(1-\sigma)^{M-k-1} P_s(n), \\ & k > j. \end{cases} \tag{8}$$

Let us now focus on the state of the system at the imbedded slots defined above. Clearly N^{t_e} constitutes an imbedded Markov chain. Let $P = (p_{nk})$ be its transition matrix; that is

$$p_{nk} \triangleq \Pr\{N^{t_e+I+T+1} = k / N^{t_e} = n\}. \tag{9}$$

To compute P, we first compute the matrix $P' = (p_{nj}')$ defined by

$$p_{nj}' \triangleq \Pr\{N^{t_e+I+T} = j / N^{t_e} = n\}. \tag{10}$$

It is clear that P' is computed by

$$P' = RQ^T \tag{11}$$

where Q^T is the Tth power of the matrix Q. The matrix P is then simply computed as follows:

$$p_{nk} = \sum_{j=n}^{M} p_{nj}' q_{jk}'(n). \tag{12}$$

From a practical point of view, it is advantageous to realize that P can be more easily computed from matrix $P'' = (p_{ij}'') = RQ^{T+1}$ by the following simple transformation:

$$p_{nk} = p_{nk}''[1 - P_s(n)] + p_{n,k+1}'' P_s(n). \tag{13}$$

Given M (finite), σ and ν, the finite state imbedded Markov chain is ergodic and a stationary probability distribution exists. We denote the latter by

$$\Pi = \{\pi_0, \cdots, \pi_j, \cdots, \pi_M\}$$

where

$$\pi_j = \lim_{t_e \to \infty} \Pr\{N^{t_e} = j\}. \tag{14}$$

Π is obtained through the system of equations

$$\Pi = \Pi P. \tag{15}$$

Distribution of the Length of the Idle Period

Let $\eta_k(i) \triangleq \Pr\{I = k / N^{t_e} = i\}$. Since the state of the system remains unchanged during the idle period, $\delta_i = (1-\nu)^i(1-\sigma)^{M-i}$ is the probability, given $N^t = i$, that no terminals become ready during slot t; thus we have

$$\eta_k(i) = (1-\delta_i)\delta_i^{k-1}. \tag{16}$$

The average idle period is $1/(1-\delta_i)$.

Stationary Average Channel Backlog

Let $\overline{N}$ denote the overall time average of the channel backlog. $\overline{N}$ is given by

$$\overline{N} = \frac{\displaystyle\sum_{i=0}^{M} \pi_i \left[\frac{1}{1-\delta_i} i + A(i)\right]}{\displaystyle\sum_{i=0}^{M} \pi_i \left[\frac{1}{1-\delta_i} + T + 1\right]} \tag{17}$$

where

$$A(i) = \sum_{m=0}^{T} \sum_{j=i}^{\min(M, i+m+1)} j s_{ij}^{(m)} \tag{18}$$

and $s_{ij}^{(m)}$ is the (i, j)th element of matrix $S^{(m)}$ defined by

$$S^{(m)} \triangleq RQ^m, \qquad 0 \leq m \leq T. \tag{19}$$

Proof: Consider a period of time consisting of a large number L of cycles during which the channel is assumed to have already reached steady-state. By definition, $\overline{N}$ can be obtained as the ratio of the sum of backlogs over all slots in the period to the length (in slots) of that period. For this, we first recognize that $\pi_i L$ is the expected number of cycles such that the backlog at their imbedded points, N^{t_e}, equals i. $S^{(m)} = (s_{ij}^{(m)})$ as defined in Eq. (19) is the transition probability matrix of the process up to the mth slot following the start of transmission of the packet. For each of these cycles, the sum of backlogs over the transmission period is given by

$$A(i) = \sum_{m=0}^{T} \sum_{j=i}^{\min(M, i+m+1)} j s_{ij}^{(m)}.$$

If the length I of the idle period is equal to k slots, then ki is the sum of backlogs over the idle period. By taking the sum over the idle periods of all cycles such that $N^{t_e} = i$, we see that, letting L be arbitrarily large, this sum is, by renewal theory arguments, equal to $\pi_i L[1/(1-\delta_i)]i$. Therefore, the total sum of backlogs is simply expressed as

$$\sum_{i=0}^{M} \pi_i L \left[\frac{1}{1-\delta_i} i + A(i) \right].$$

By a similar argument, the length of the period of L cycles is given by

$$\sum_{i=0}^{M} \pi_i L \left[\frac{1}{1-\delta_i} + T + 1 \right].$$

By taking the ratio and letting $L \to \infty$, we get $\bar{N}$ as expressed in Eq. (17). *Q.E.D.*

Stationary Average Channel Throughput

The average normalized throughput (defined as the average number of successful packets per T slots), denoted by S_{out}, is given by

$$S_{\text{out}} = \frac{\sum_{i=0}^{M} \pi_i T P_s(i)}{\sum_{i=0}^{M} \pi_i \left[\frac{1}{1-\delta_i} + T + 1 \right]} \tag{20}$$

where $P_s(i)$ is givenby Eq. (7).

Proof: $P_s(i)$ is the probability of a successful transmission over a cycle such that $N^{te} = i$. By using arguments similar to the previous proof, we get Eq. (20). *Q.E.D.*

Expected Packet Delay

The expected packet delay is also the average backlog time; by Little's result [8], it is simply expressed as

$$D = \frac{\bar{N}}{S_{\text{out}}}. \tag{21}$$

Some Computational Considerations

The numerical evaluation of the above model leads to the (equilibrium) throughput-delay performance of a slotted nonpersistent CSMA mode in a finite population environment. However, for large systems (large T, large M), serious computational problems occur, causing an enormous amount of underflows to take place in the numerical computation of $\boldsymbol{P}$. Indeed, a typical value for T (typical for the ground radio systems in consideration) is 100. On the other hand, the throughput S_{out} is always upper bounded by 1 and, as we shall see in the following section, the value of $M\sigma T$ which leads to a *reasonable* channel operating behavior is also on the order of 1. Therefore, we see that the maximum value of σ (probability that a thinking terminal generates a new packet in a time-slot) is extremely small (for $T = 100$ and $M = 100$, for example, $\sigma \cong 10^{-4}$) causing the underflows to occur. Fortunately, approximations are possible which permit us to alleviate the computational difficulties without seriously affecting the numerical results. For such a small σ, we shall approximate the binomial distribution of the number of new packets generated per slot from the pool of thinking users by a Bernouilli distribution. (The approximation has been checked on small size systems, $T = 10$, $M = 20$, to give extremely accurate results. For $T = 100$, the two models are essentially identical!) Therefore, for the purpose of this study, we shall use the following simplified forms for various previously defined quantities:

$$r_{ik} \cong \begin{cases} 0, & k < i \\ 1 - \dfrac{(M-i)\sigma}{1-(1-\nu)^i[1-(M-i)\sigma]}, & k = i \\ \dfrac{(M-i)\sigma}{1-(1-\nu)^i[1-(M-i)\sigma]}, & k = i+1 \\ 0, & k > i+1 \end{cases}$$

$$q_{ik} \cong \begin{cases} 0, & k < i \\ 1-(M-i)\sigma, & k = i \\ (M-i)\sigma, & k = i+1 \\ 0, & k > i+1 \end{cases}$$

$$q_{ik}'(n) \cong \begin{cases} 0, & k < i-1 \\ [1-(M-i)\sigma]P_s(n), & k = i-1 \\ [1-(M-i)\sigma]\,[1-P_s(n)] \\ \quad + (M-i)\sigma P_s(n), & k = i \\ (M-i)\sigma[1-P_s(n)], & k = i+1 \\ 0, & k > i+1 \end{cases}$$

$$P_s(n) \cong \frac{(1-\nu)^n(M-n)\sigma + n\nu(1-\nu)^{n-1}[1-(M-n)\sigma]}{1-(1-\nu)^n[1-(M-n)\sigma]}.$$

Moreover, we show in Appendix A that due to the special (almost triangular) form of matrix $\boldsymbol{P}$, characterized by $p_{ij} = 0$ for $j \leqslant i - 2$. the solution of the system given in Eq. (15) may easily be obtained recursively. This recursive method is such that each column of $\boldsymbol{P}$ is used exactly once and can be generated only when needed. This eliminates a major computer storage constraint on the dimensionality of the problem which would be encountered were it necessary to store the entire matrix at once.

Some Numerical Results

Using the finite population model described in Section 2, we can analytically verify the conclusions regarding the effect of retransmission delay on channel performance drawn from simulation in Part I [1]. For this we plot in Fig. 2, the delay-throughput curves for $M = 50$ and various values of ν. The average retransmission delay here is $1/\nu$ slots. We note that for decreasing values of ν the maximum achievable average throughput increases. Moreover, for each value of S_{out}, there exists a value of ν which achieves minimum delay. The throughput-delay performance of the system is obtained by taking the lower envelope of these curves. Fig. 3 displays these lower envelopes for various values of M (In the case $M = 5000$, due to excessive computation, the search for the lower envelope was not carried out to the full extent; the anticipated lower envelope is shown as a dashed line.)

4. STABILITY CONSIDERATIONS

Theory

The following treatment strongly follows the work by Lam and Kleinrock on slotted ALOHA channels [3, 4].

Consider an arbitrary cycle such that $N^{te} = n$. The expected number of successful packets during the cycle is given by $P_s(n)$ (see Eq. (7)). On the other hand, the expected number of new packets generated in the system is the expected increase in N^t over the entire cycle; it is expressed as

$$S_{\text{in}}(n, \sigma) = \frac{(M-n)\sigma}{1-(1-\nu)^n(1-\sigma)^{M-n}} + \left[M - \frac{A(n)}{T+1}\right]\sigma(T+1) \tag{22}$$

where $A(n)$ is the sum of backlogs over the transmission period and is given by Eq. (18).

Let $\overline{S_{\text{in}}(n, \sigma)}$ be the average input rate over the cycle normalized with respect to T; it is expressed as

$$\overline{S_{\text{in}}(n, \sigma)} = \frac{S_{\text{in}}(n, \sigma)T}{1/(1-\delta_n)+T+1} \tag{23}$$

where $[1/(1-\delta_n)] + T + 1$ is the average cycle length. Given M, σ and ν, $\overline{S_{in}(n, \sigma)}$ is also referred to as the average instantaneous *channel load*. For fixed values of ν and σ, given $N^{te} = n$, we plot n versus $P_s(n)$ and $S_{in}(n, \sigma)$. Examples are shown in Fig. 4. (Note that $P_s(n)$ is insensitive to variations in σ, this was successfully tested for a wide range of M and σ.) We note here that there are equilibrium points defined (at some value of n) by $P_s(n) = S_{\text{in}}(n, \sigma)$. On the other hand, for each value of n, we can find a value σ^* of σ such that $P_s(n) = S_{\text{in}}(n, \sigma^*)$. Consider now the two-dimensional plane (n, S) on which we plot $\overline{S_{\text{in}}(n, \sigma^*)}$ versus n. This plot determines an *equilibrium contour* [3, 4] defined by the property that, for each n, the expected number of successful packets over the cycle equals the expected number of packets newly generated during that cycle. For a given ν and a given n, $P_s(n)$ decreases and $S_{\text{in}}(n, \sigma)$ (and therefore $\overline{S_{\text{in}}(n, \sigma)}$) increases for increasing values of σ. Thus for $\sigma > \sigma^*$, $S_{\text{in}}(n, \sigma)$ will exceed $P_s(n)$. This means that in the region enclosed by the equilibrium contour, the ex-

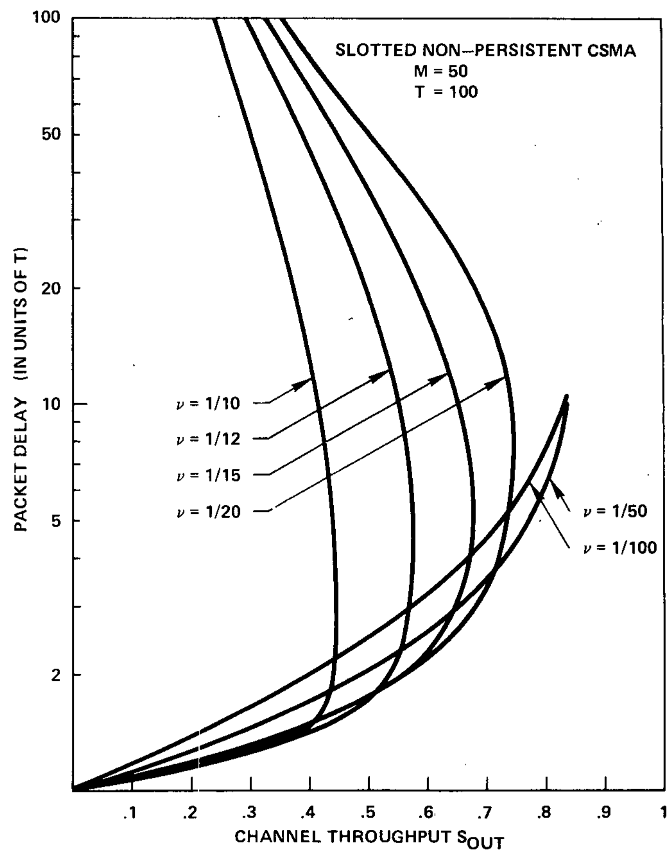

Fig. 2. Delay-Throughput Curves in Finite Population Slotted Nonpersistent CSMA Channel.

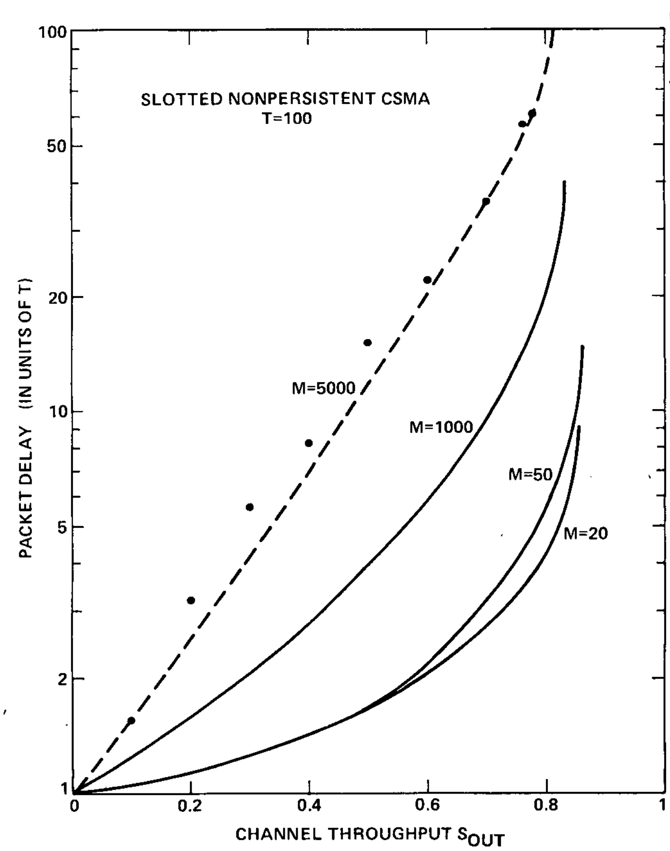

Fig. 3. Slotted Nonpersistent CSMA Channels: Minimum delay for Various Values of $M(T = 100)$.

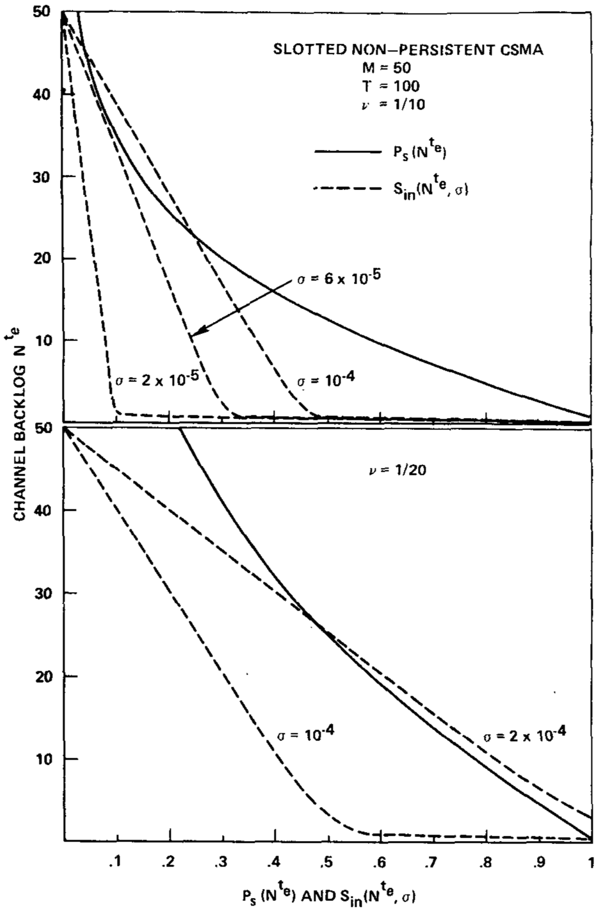

Fig. 4. Expected Number of Successful Packets $P_s(n)$ and New Packets $S_{in}(n, \sigma)$.

pected number of successful packets $P_s(n)$ exceeds the expected number of newly generated packets; elsewhere, the opposite is true and the system capacity is exceeded! Using σ^* for each n, we may plot a family of equilibrium contours for various values of ν. To simplify this task, we note that some approximations are possible.

We first observe that consistently $[A(n)/(T+1)]$ is very close to n so that it is legitimate to approximate $M - [A(n)/(T+1)]$ by $M - n$. (In fact, the only significant discrepancy between $[A(n)/(T+1)]$ and n is seen for small n, namely, $n = 0$ and $n = 1$, which of course has little effect on $M - [A(n)/(T+1)]$.) Under this approximation, $S_{in}(n, \sigma)$ and $\overline{S_{in}(n, \sigma)}$ can be approximated by

$$S_{in}(n, \sigma) \cong S_{in}'(n, \sigma) = (M - n)\sigma \left[\frac{1}{1-\delta_n} + T + 1\right] \quad (24)$$

$$\overline{S_{in}(n, \sigma)} \cong \overline{S_{in}'(n, \sigma)} = (M - n)\sigma T \quad (25)$$

(an intuitively pleasing result). For the infinite population model, i.e., in the limit as $M \uparrow \infty$ and $\sigma \downarrow 0$ such that $M\sigma = s$ is finite and the channel input is Poisson distributed at the constant rate s (packets per slot), the above equations reduce to

$$P_s(n) \cong P_s'(n) = \frac{(1-\nu)^n s e^{-s} + n\nu(1-\nu)^{n-1} e^{-s}}{1-(1-\nu)^n e^{-s}} \quad (26)$$

$$S_{in}(n, \sigma) \cong S_{in}'(n, s) = \frac{s}{1-(1-\nu)^n e^{-s}} + s(T+1). \quad (27)$$

These expressions are very accurate even for finite M if $\sigma \ll 1$ and if we replace $s = M\sigma$ by $s = (M - n)\sigma$. (The condition $\sigma \ll 1$ is always satisfied in problems of interest to this study; see our comments on computational considerations above.) Equating $P_s'(n)$ to $S_{in}'(n, \sigma)$ we can determine for each n the value of s and therefore sT ($\cong (M - n)\sigma T \cong \overline{S_{in}'(n, \sigma)}$) which defines the equilibrium contours. In fact, since for each point of the equilibrium contour equality holds between $P_s'(n)$ and $S_{in}'(n, \sigma)$, it is equivalent to searching for that value of s at which the input rate sT equals the instantaneous throughput, denoted by $\overline{S_{out}(n)}$ and defined as

$$\overline{S_{out}(n)} \triangleq \frac{P_s(n)T}{1/(1-\delta_n) + T + 1}. \quad (28)$$

The solution is obtained by solving for s in the following nonlinear equation

$$(1-\nu)^n \left[(T+2)(1-\nu)se^{-s} + n\nu e^{-s}\right] = s(T+2). \quad (29)$$

(The accuracy obtained by using these approximate expressions has been successfully verified by comparing results using both sets of equations, exact and approximate.) In Fig. 5, the family of equilibrium contours for various ν are displayed. We see that if we increase the average retransmission delay (by decreasing ν), these equilibrium contours move upwards.

We have so far defined the equilibrium contours based on a *fluid approximation* interpretation; that is, we let the *expected* number of successful packets over a cycle, $P_s(n)$, be equal to the expected number of newly generated packets over the cycle, $S_{in}(n, \sigma)$. The direction of fluid flow is simply determined by the tendency the system has to increase the channel backlog (and this occurs when $P_s(n) < S_{in}(n, \sigma)$, i.e., the point $(n, \overline{S_{in}(n, \sigma)})$ lies outside the equilibrium contour) or to decrease the channel backlog ($P_s(n) > S_{in}(n, \sigma)$, i.e., the point $(n, \overline{S_{in}(n, \sigma)})$ lies inside the equilibrium contour). Note that these equilibrium contours have the same shape as those encountered in slotted ALOHA channels [3, 4].

Under the approximation that $[A(n)/(T+1)] \cong n$, we have

$$\overline{S_{in}(n, \sigma)} \cong \overline{S_{in}'(n, \sigma)} = (M - n)\sigma T. \quad (30)$$

As in [4], given M and σ, we define the *channel load line* in the (n, S) plane as the line $S = (M - n)\sigma T$ which intercepts the n-axis at $n = M$ and has a slope equal to $-(1/\sigma T)$. Stability here is also defined as in [4] and [9]. We quote their discussion below.

> "A channel load line may intersect (nontangentially) the equilibrium contour one or more times, and we refer to these as equilibrium points denoted by (n_e, S_e). An equilibrium point on a load line is said to be a *stable equilibrium point* if it acts as a 'sink' with respect to the drift of N^t; it is said to be an *unstable equilibrium point* if it acts as a 'source'. A stable equilibrium point is said to be a *channel operating point* if $n_e \leqslant n_{max}$ as shown

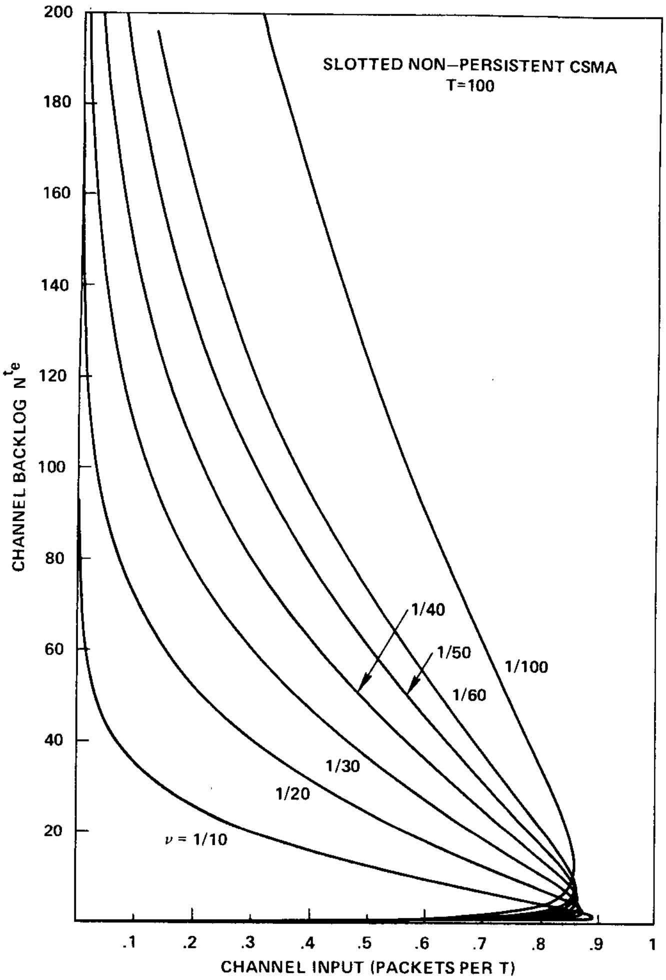

Fig. 5. Equilibrium Contours.

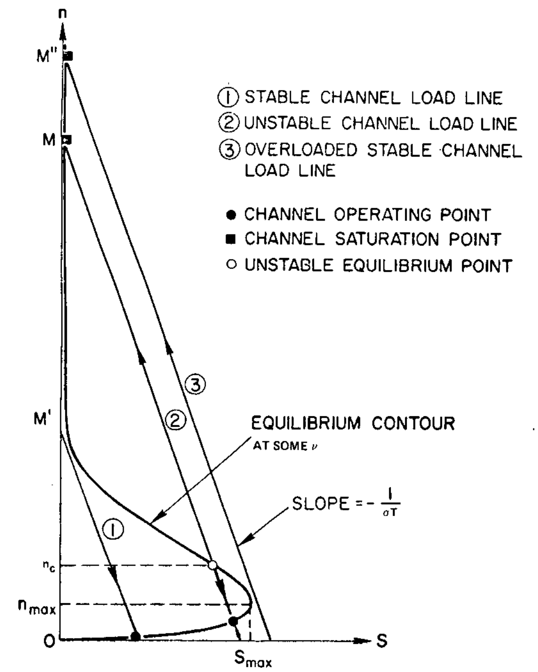

Fig. 6. Hypothetical Stable and Unstable Channels [9].

in Fig. 6. It is said to be a *channel saturation point* if $n_e > n_{\max}$. (We shall used (n_0, S_0) instead of (n_e, S_e) to distinguish a channel operating point from other equilibrium points.) A channel load line is said to be *stable* if it has exactly one stable equilibrium point; otherwise, it is said to be *unstable.* Thus the load lines labeled 1 and 3 in Fig. 6 are stable by definition; the load line labled 2 is unstable. In a stable channel, the equilibrium point (n_e, S_e) determines the steady-state throughput-delay performance of the channel over an infinite time horizon. On the other had, an unstable channel exhibits 'bistable' behavior; the throughput-delay performance given by the channel operating point is achievable only for a finite time period before the channel drifts towards the channel saturation point. When this happens, the channel performance degrades rapidly as the channel throughput rate decreases and the average packet delay increases. The channel load line labeled 3 in Fig. 6 has a channel saturation point as its only stable equilibrium point. It is overloaded in the sense that M'' is too big for the given σ and ν. From now on, a stable channel load line will always refer to 1 instead of 3."

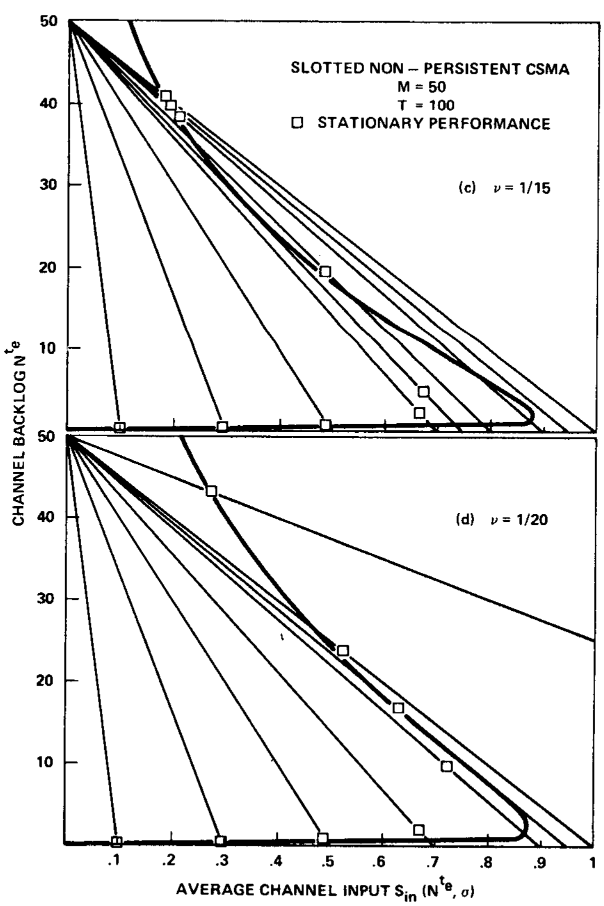

Fig. 7. Examples of Actual Stable and Unstable Channels in Slotted Nonpersistent CSMA.

In Figs. 7(a, b) we show actual load lines corresponding to the above cases; the stationary channel performance $(\overline{N}, S_{\text{out}})$ as calculated from Eqs. (17) and (20) is shown on each of the load lines. We note for stable channels that the average stationary performance coincides with the operating point (n_0, S_0), while for overloaded channels, it coincides with the channel saturation point; for unstable channels, the average stationary performance lies between the operating point and

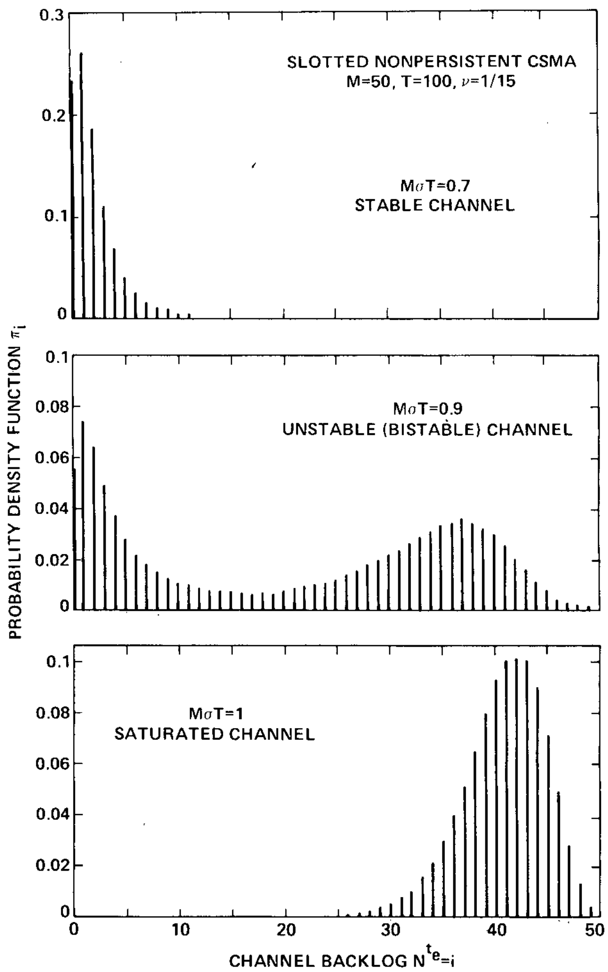

Fig. 8. Stationary Backlog Distribution for Stable, Unstable, and Saturated Channels.

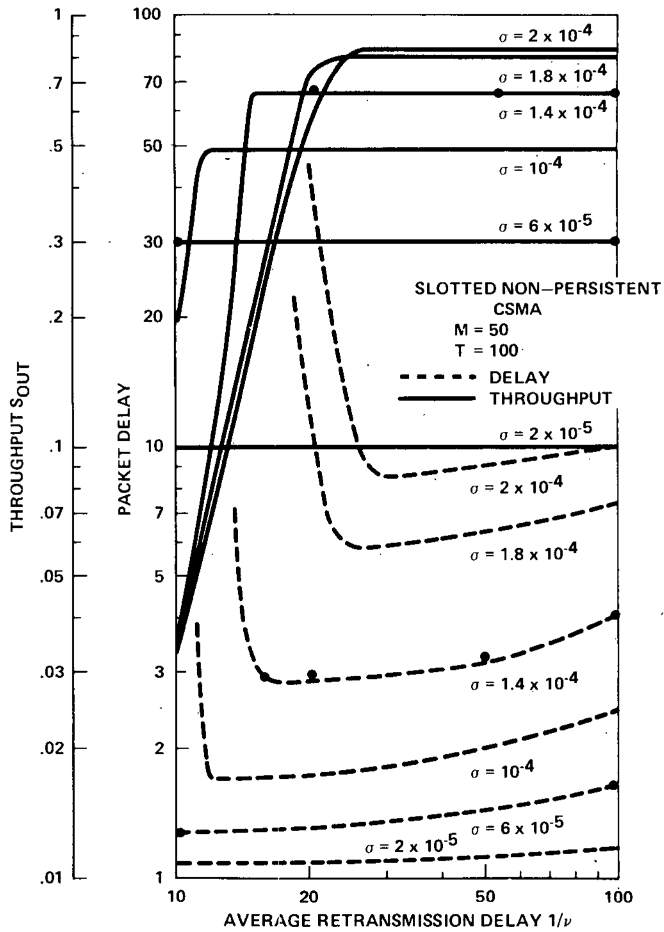

Fig. 9. Channel Performance versus $1/\nu$ (Dots Represent Simulation Points).

the saturation point. These statements are verified by observing that, as displayed in Fig. 8, the density function of the stationary backlog distribution Π obtained by analysis in Section 3 is concentrated around n_0 for a stable channel, is concentrated around the saturation point for an overloaded channel, and exhibits the bistable behavior for unstable channels.

As in slotted ALOHA channels [4], we also see that as the number of channel users M increases, an originally stable channel becomes unstable although the channel input rate S_0 at the operating point may be made to remain constant (by reducing σ). In Fig. 9, we show that an originally unstable channel can be rendered stable by selecting ν smaller than a critical value, below which the system performance is excellent and only slightly sensitive to ν. (Some simulation results of the feedback model for $M = 50$ are also shown in Fig. 9. Results from both analysis and simulation agree beautifully.)

The channel load line of an infinite population model is a vertical line. This is always an unstable channel according to the stability definition. (Note that $N = \infty$ is a stable equilibrium point; i.e., the channel saturation point.) In fact, since N^{te} has an infinite state space and $S_{\text{in}}(n, \sigma) > P_s(n)$ for $n > n_c$, where n_c is the unstable equilibrium point (see Fig. 6), a stationary probability distribution *does not exist* for N^{te}. (See, for example, Cohen [10, pp. 543-546], for such a proof.)

FET: A Stability Measure

Kleinrock and Lam [4] extended these stability considerations by defining a stability measure for unstable channels. For this they considered the load line of an unstable channel to be partitioned into two regions: the *safe* region consisting of the system states $\{0, 1, 2, \cdots, n_c\}$ and the *unsafe* region consisting of the system states $\{n_c + 1, \cdots, M\}$. The stability measure refers to the average time to exit into the unsafe region starting from a safe channel state. More precisely, they define FET to be the average *first exit time* into the unsafe region starting from an initially empty channel ($N^{te} = 0$). Thus, FET gives an approximate measure of the average uptime of an unstable channel.

The derivations of FET are based upon well-known results of first passage times in the theory of Markov chains with stationary transition probabilities [11, 12]. (The derivations apply to the infinite M case as well.) Consider the modified state space $\{0, 1, 2, \cdots, n_c, n_u\}$ where n_u is an absorbing state obtained by merging the entire unsafe region into a single state n_u with the transition probability $P_{n_u,n_u} = 1$. Define the random variable T_i as the number of transitions which N^{te}

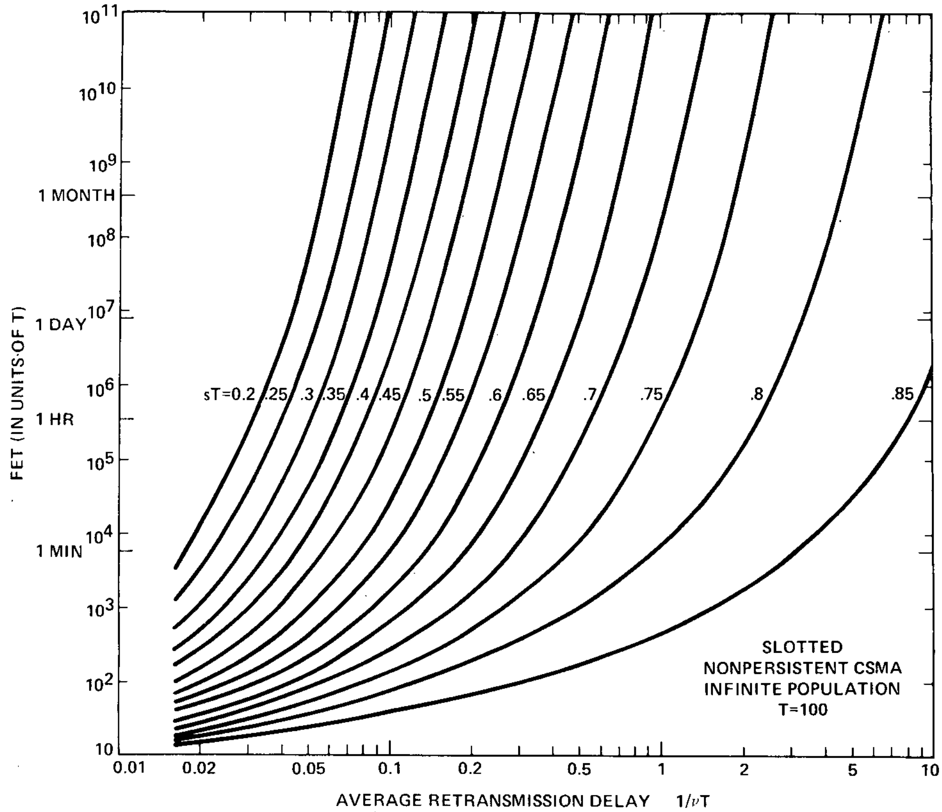

Fig. 10. FET Values for the Infinite Population Model (T = 100).

goes through before it enters the unsafe region for the first time starting from state i in the safe region. Let $\overline{T}_i$ be the expected value of T_i. For all i ($0 \leqslant i \leqslant n_c$), it is easily shown [11] that

$$\overline{T}_i = \begin{cases} \dfrac{1}{1-\delta_i} + T + 1, & \text{with probability } \displaystyle\sum_{l=n_c+1}^{M} p_{il} \\ \dfrac{1}{1-\delta_i} + T + 1 + \overline{T}_j, & \text{with probability } p_{ij} \end{cases} \quad (31)$$

This leads to the following set of $n_c + 1$ linear simultaneous equations which $\{T_i\}_{i=0}{}^{n_c}$ must satisfy:

$$\overline{T}_i = \frac{1}{1-\delta_i} + T + 1 + \sum_{j=0}^{n_c} p_{ij}\overline{T}_j, \qquad 0 \leqslant i \leqslant n_c. \quad (32)$$

The stability measure FET is actually $\overline{T}_0$. Given the fact that $p_{ij} = 0$ for $j \leqslant i - 2$, we can use a recursive algorithm derived in [4, Appendix] to solve the above system. This algorithm is very efficient in terms of space requirements in that at each iteration it requires only a single row of the transition matrix and uses each element in the row exactly once. (This algorithm is given in Appendix B.) As applied to our case, a considerable saving in storage is obtained by realizing that, as shown in Appendix B, individual rows of P can be computed separately.

In Fig. 10, we show FET as a function of the average retransmission delay, $1/\nu T$, for the infinite population model and for fixed values of the channel throughput rate sT. FET can be improved by increasing $1/\nu T$, which in turn increases the average packet dleay. Fig. 11 displays similar curves for the M = 1000 case. Note the improvement in FET for the finite population.

Channel Performance During the Uptime of Unstable Channels

Initiating an unstable channel (M, σ, ν) at time $t = 0$ in the zero state, the system remains in the safe region for a period of time equal to T_0. At time T_0, the backlog reaches $n_c + 1$; since for $N^{te} > n_c$ the drift on the load line is in the direction of increasing backlog, the channel is considered to have failed, although it has a non-zero probability of returning to the safe region; in practical situations, we assume that the channel is restarted anew with a zero backlog whenever it "fails." Operating the system in this mode, the question of interest is: what is the average delay of those packets successfully transmitted during the uptime of the channel?

Consider the (reducible) imbedded Markov chain on the modified state space $\{0, 1, 2, \cdots, n_c, n_u\}$ described above and diagrammed in Fig. 12(a). We model the cold restart of the channel every time it fails by allowing a return transition of probability 1 from state n_u back to state 0 (see Fig. 12(b)). N^{te} is now a finite irreducible Markov chain characterized by the transition matrix $P^{(m)} = (p_{ij}{}^{(m)})$ given by

$$p_{ij}{}^{(m)} = \begin{cases} p_{ij}, & i, j = 0, 1, \cdots, n_c \\ \displaystyle\sum_{l=n_c+1}^{M} p_{il}, & i = 0, 1, \cdots, n_c; \quad j = n_u \\ 0, & i = n_u; \quad j = 1, 2, \cdots, n_u \\ 1, & i = n_u; \quad j = 0. \end{cases} \quad (33)$$

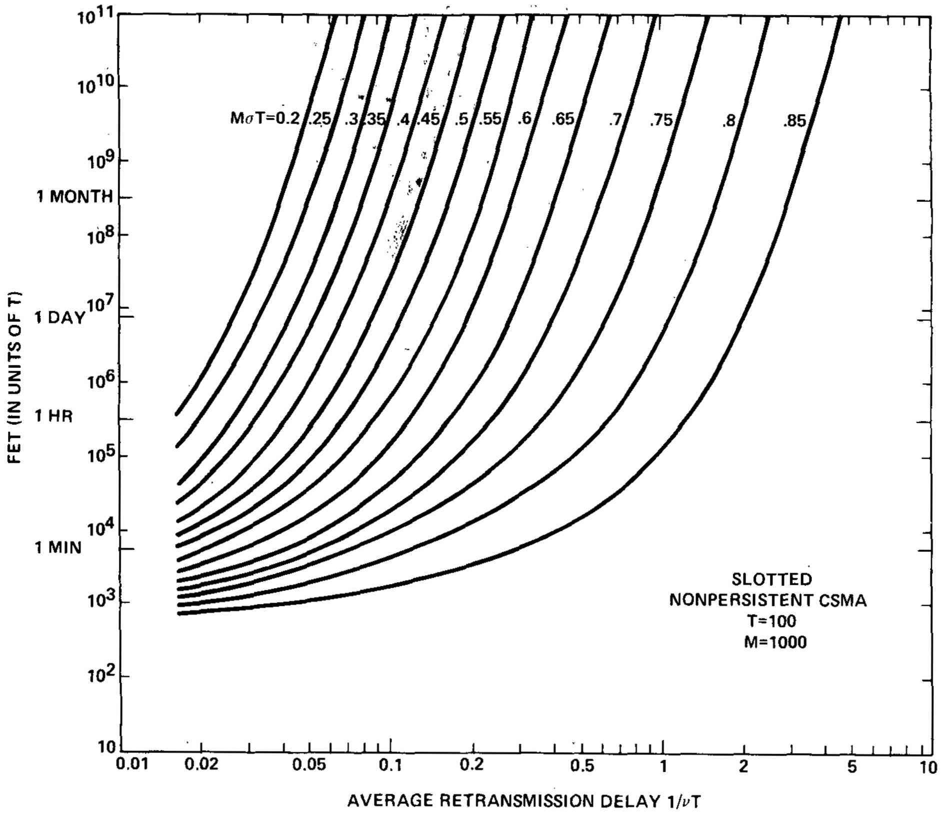

Fig. 11. FET Values for a Finite User Population (M = 1000, T = 100).

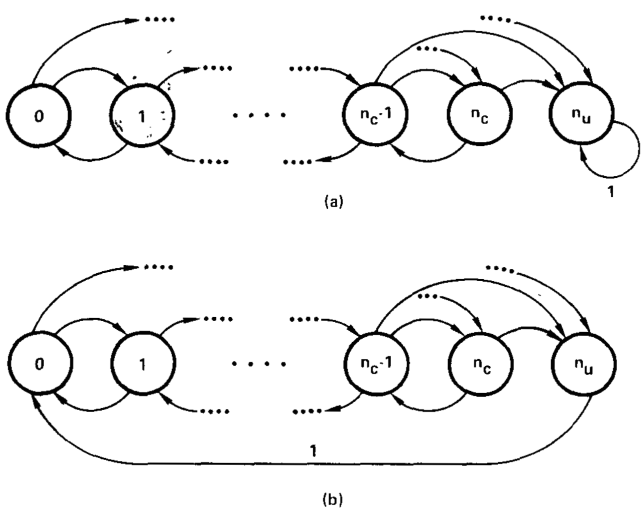

Fig. 12. Markov Chain State Diagrams. (a) With an Absorbing State n_u. (b) Modeling Cold Restarts.

Let $\Pi^{(m)} = \{\pi_1{}^{(m)}, \pi_2{}^{(m)}, \cdots, \pi_{n_u}{}^{(m)}\}$ be the stationary probability distribution. $\Pi^{(m)}$ is the solution of the system

$$\Pi^{(m)} = \Pi^{(m)}P^{(m)}. \tag{34}$$

Despite the presence of $p_{n_u,0} = 1$ which violates the original property $p_{ij} = 0, j \leqslant i - 2$, we show in Appendix C that there is still a recursive method to solve system (34).

Given $\Pi^{(m)}$, the packet delay $D^{(m)}$ (averaged over all successful packets) during the uptime of unstable channels is given by

$$D^{(m)} = \frac{\displaystyle\sum_{i=0}^{n_c} \pi_i{}^{(m)} \left[\frac{1}{1-\delta_i} i + A(i)\right] + \pi_{n_u}{}^{(m)}A(0)}{\displaystyle\sum_{i=0}^{n_c} \pi_i{}^{(m)}TP_s(i) + \pi_{n_u}{}^{(m)}TP_s(0)}. \tag{35}$$

Proof: The average packet delay is obtained as the ratio of the time integral of the backlog over a long period of time to the number of successful packets over that period. Consider a period of time consisting of a large number L of cycles where $L \gg$ FET. The time integral of the backlog over the period is given by

$$L\left[\sum_{i=0}^{n_c} \pi_i{}^{(m)} \left[\frac{1}{1-\delta_i} i + A(i)\right] + \pi_{n_u}{}^{(m)}A(0)\right].$$

The number of successful packets during the L cycles is given by

$$L\left[\sum_{i=0}^{n_c} \pi_i{}^{(m)}P_s(i) + \pi_{n_u}{}^{(m)}P_s(0)\right].$$

Taking the ratio, dividing by T and letting $L \to \infty$, we get the normalized delay $D^{(m)}$ as expressed in Eq. (35).‡ *Q.E.D.*

In Fig. 13 we show the throughput-delay performance for the slotted ALOHA and CSMA infinite population models

‡A simpler expression is obtained by approximating $A(i)$ by $(T + 1)i$ for $i \geqslant 1$ and $A(0)$ by $T + 1$.

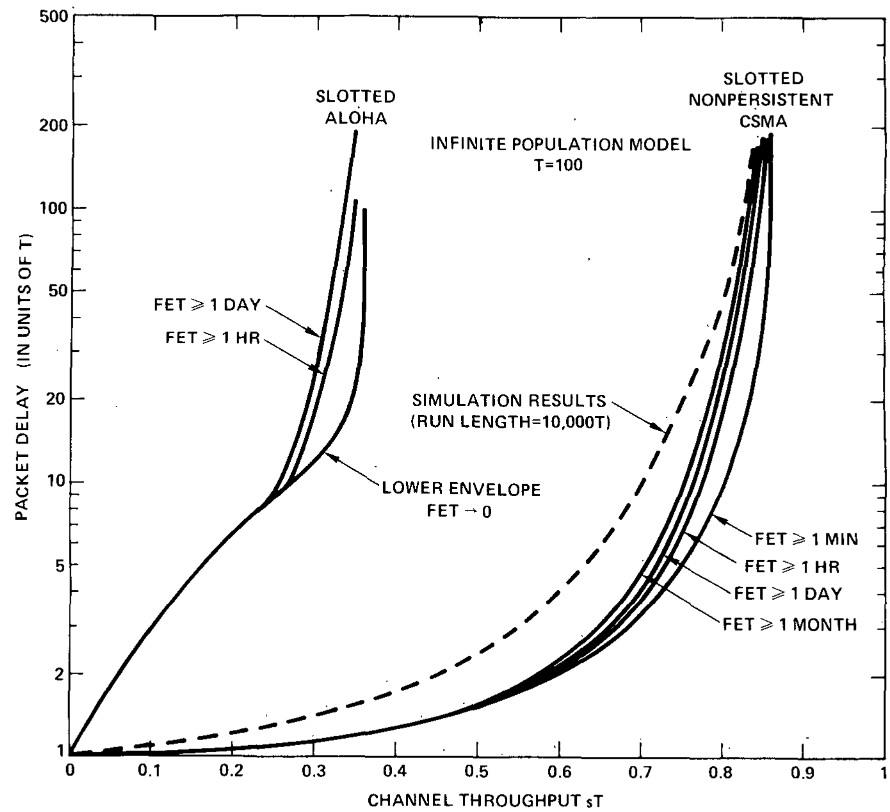

Fig. 13. Slotted ALOHA and Nonpersistent CSMA: Stability-Delay-Throughput Tradeoff for the Infinite Population Model ($T = 100$).

with guaranteed FET of 1 minute, 1 hour, 1 day and 1 month. We note that the degradation in performance due to increasing FET from 1 minute to 1 month is not very significant. Although the delay is theoretically infinite for FET $= \infty$, the delay degradation due to increasing FET becomes less important with larger FET so that very good performance can practically be achieved over extremely long periods of time. This is easily explained by observing that the FET curves for constant S in Fig. 10 become very steep with increasing retransmission delays. We also show in Fig. 13 the throughput-delay results for the infinite population as obtained in Part I [1] by simulation; we note that these results were a rather conservative prediction of the true performance of CSMA channels for a period ot time of at least 1 month. Fig. 14 displays similar results for the $M = 1000$ case; the FET $= \infty$ throughput-delay curve is obtained through the steady-state analysis of Section 3. Corresponding results for slotted ALOHA are also shown on both Figs. 13 and 14 for comparative purposes; the comparison demonstrates once more the clear superiority of CSMA over slotted ALOHA.

5. DYNAMIC CONTROL FOR IMPROVED PERFORMANCE IN CSMA CHANNELS

If M is finite, a stable channel can always be achieved by using a sufficiently small ν. Of course, a small ν implies that the equilibrium backlog size is large and hence the average packet delay is also large. We also note for a stable channel that, as M increases, the packet delay increases for the same total throughput achieved, S_0. This is best seen in Fig. 15 where we plot packet delay versus M for fixed throughput. Dynamic control procedures can be applied which will enable an originally unstable channel to achieve a throughput-delay performance close to its desired operating point with guaranteed stability, i.e., over an infinite time horizon. Conversely, these procedures can be applied in order to improve the (high-) delay performance of a stable channel with large M. In the context of slotted ALOHA, two classes of control actions were considered in [9]; namely, the input control and the retransmission control. The input control procedure allows the channel to either accept or reject new packets from their sources. The retransmission control procedure allows the channel transmitters to impose either large or small retransmission delays on previously collided packets. For the purpose of here illustrating the improvement in the performance and stability of CSMA due to control, we limit ourselves to the second class of control.

A Dynamic Retransmission Control Policy

Given $N^{te} = n$, the instantaneous throughput over the cycle $\overline{S_{\text{out}}(n)}$ was defined in Eq. (28) and is explicitly expressed for the finite M case as

$$\overline{S_{\text{out}}(n)} = \frac{(1-\nu)^n (M-n)\sigma T + n\nu(1-\nu)^{n-1}[1-(M-n)\sigma]T}{1+(T+1)[1-(1-\nu)^n(1-(M-n)\sigma)]} \tag{36}$$

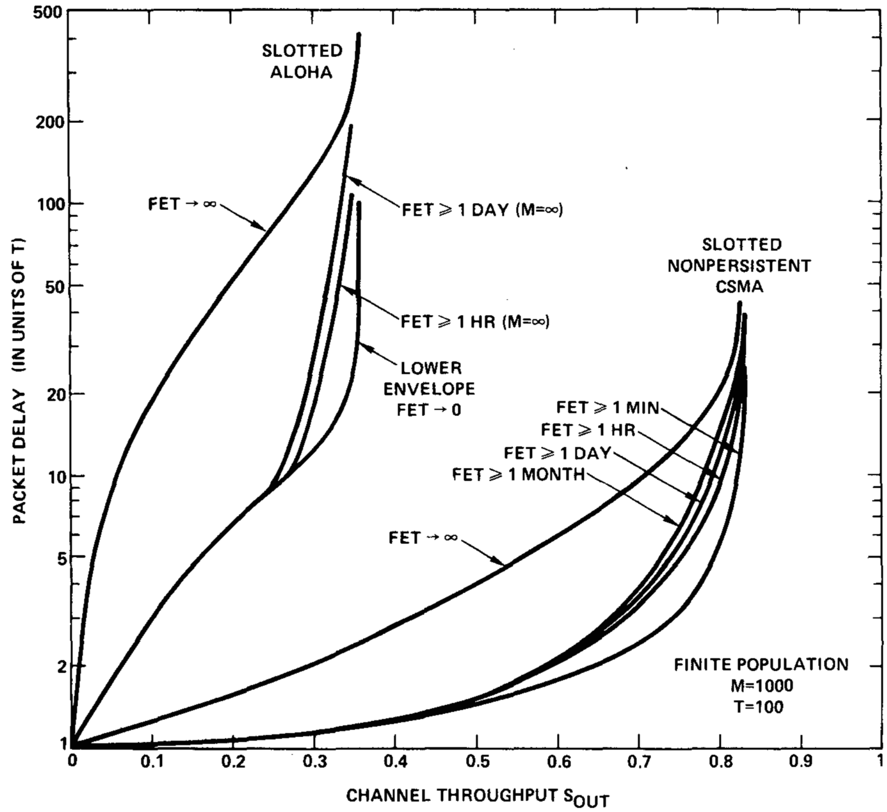

Fig. 14. Slotted ALOHA and Nonpersistent CSMA: Stability-Delay-Throughput Tradeoff for a Finite Population ($M = 1000$, $T = 100$).

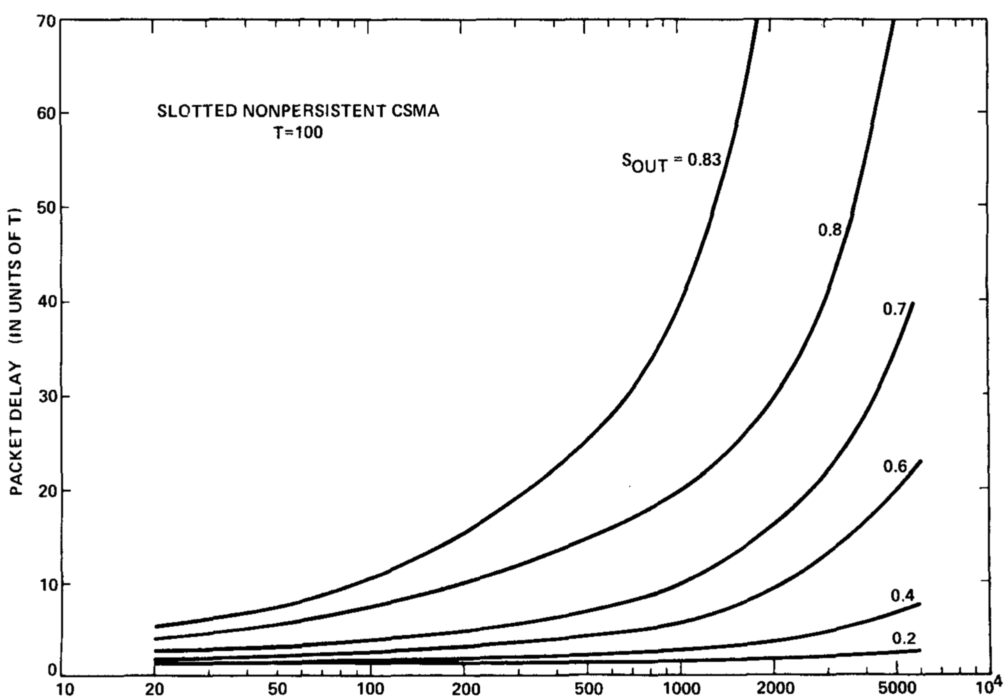

Fig. 15. Slotted Nonpersistent CSMA: Steady-State Packet Delay versus $M (T = 100)$.

and for the infinite population case as

$$\overline{S_{\mathrm{out}}(n)} = \frac{(1-\nu)^n s e^{-s} T + n\nu(1-\nu)^{n-1} e^{-s} T}{1 + (T+1)[1-(1-\nu)^n e^{-s}]}. \tag{37}$$

$\overline{S_{\mathrm{out}}(n)}$ is a function of ν and is characterized by the fact that, for fixed ν, it goes to zero with increasing values of n. *Assuming* now that each user knows the exact state of the system at the beginning of a cycle,§ then one can improve the performance by maximizing the instantaneous throughput over the retransmission probability ν. Let $\nu^*(n)$ denote this optimum. We shall seek a *simple* expression for $\nu^*(n)$ and study the effect of applying this dynamic control on the stability and the overall performance of CSMA channels.

The above expressions for $\overline{S_{\mathrm{out}}(n)}$ are complex and depend on several system parameters, namely M, σ, and T; an exact optimization of $\overline{S_{\mathrm{out}}(n)}$ is untractable. Fortunately, some approximations are again possible here which significantly reduce the complexity of the problem. First, we know that Eq. (37) is a very accurate expression, even for finite M, as long as we take $s = M\sigma$. Furthermore, we observe‖ for various values of ν in the vicinity of ν^* that a very good approximation for $\overline{S_{\mathrm{out}}(n)}$ is obtained by setting $s = 0$, yielding the relatively simple expression

$$\overline{S_{\mathrm{out}}(n)} \cong \frac{n\nu(1-\nu)^{n-1} T}{1 + (T+1)[1-(1-\nu)^n]}. \tag{38}$$

Taking the derivative of Eq. (38), we see that ν^* is the solution of the following equation:

$$(T+1)y^n - (T+2)ny + (T+2)(n-1) = 0 \tag{39}$$

where $y = 1 - \nu$. Although the current problem is much simpler than the original one, it remains impractical in that we require either that each terminal be able to numerically solve Eq. (39) given n, or that each terminal contain a table for $\nu^*(n)$ as a function of n. However, further simplifications of $\overline{S_{\mathrm{out}}(n)}$ are possible by approximating $(1-\nu)^k$ by $1 - k\nu$ in Eq. (38). We get

$$S_{\mathrm{out}}(n) \cong \frac{n\nu[1-(n-1)\nu]T}{1 + (T+1)n\nu}. \tag{40}$$

§In practical situations, the assumption that each user knows the exact current state of the system clearly does not hold. The channel users have no means of communication among themselves other than the multiaccess broadcast channel itself. However, each channel user may individually *estimate* the channel state by observing the channel outcome over some period of time, and apply a control action based upon the estimate. In the context of slotted ALOHA, Lam and Kleinrock [9] give some heuristic control-estimation algorithms which prove to be very satisfactory. With appropriate modifications and extensions, these algorithms can be applied to CSMA channels as well. However, this is considered outside the scope of this paper. The results here obtained assuming full knowledge of the system state will then represent the ultimate performance; a bound on the performance obtained via heuristic estimation algorithms.

‖A numerical check has been performed for a wide range of the system parameters M, σ and T substantiating the validity of these and the following approximations.

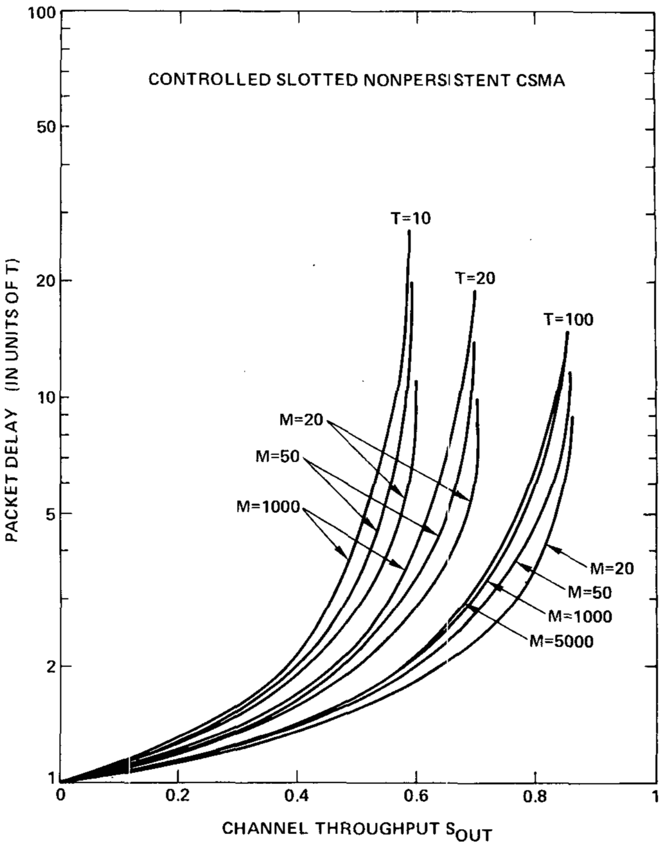

Fig. 16. Controlled Slotted Nonpersistent CSMA: Throughput-Delay Performance.

The derivative of this last yields the second degree equation

$$(T+1)n(n-1)\nu^2 + 2(n-1)\nu - 1 = 0. \tag{41}$$

The optimum ν^* (the solution of Eq. (41)) is given by

$$\nu^* = \frac{-1 + \sqrt{1 + \dfrac{n}{n-1}(T+1)}}{(T+1)n}. \tag{42}$$

Neglecting 1 as compared to T, $\sqrt{T}$ and n, we finally get the *simple* expression for ν^* given by#

$$\nu^*(n) = \begin{cases} 1, & \text{if } n = 0 \\ \dfrac{1}{n\sqrt{T}}, & \text{if } n \geqslant 1. \end{cases} \tag{43}$$

The optimum instantaneous throughput is now characterized by the fact that it reaches a non-zero limit as $n \to \infty$, namely

$$\lim_{n\to\infty} \overline{S_{\mathrm{out}}{}^*(n)} = \frac{\sqrt{a}e^{-\sqrt{a}}}{1 + 2a - (1+a)e^{-\sqrt{a}}} = C(a) \tag{44}$$

#It has been verified that this simple expression for ν^* gives an instantaneous throughput extremely close to the true maximum.

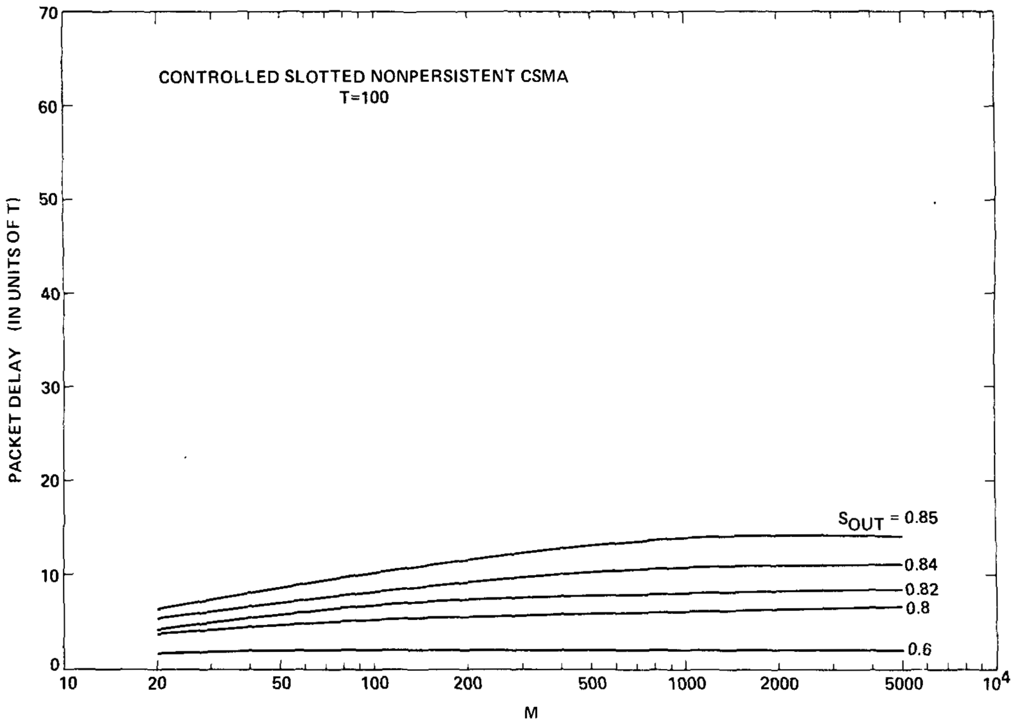

Fig. 17. Controlled Slotted Nonpersistent CSMA: Steady-State Packet Delay versus $M(T = 100)$.

where $a = 1/T$. Eq. (44) actually represents a closed form expression for the capacity of CSMA channels, since for any $n < \infty$, the average instantaneous throughput is bounded from below by $C(a)$. Essentially, the dynamic control $\nu^*(n)$ has the effect of providing an equilibrium contour consisting of basically a vertical line at an abscissa equal to $C(a)$.

The steady-state performance of a controlled CSMA channel is easily obtained from the analysis presented in Section 3, whereby row i of P is obtained by replacing ν by $\nu^*(i)$. In Fig. 16, we show the steady-state performance of controlled CSMA channels for various values of M and T. By comparing the $T = 100$ case of Fig. 16 to Fig. 3, which displays the analogous results for uncontrolled channels, we note that there is a significant improvement in delay, particularly with large populations. This is best seen in Fig. 17, which displays the packet delay for controlled channels as a function of M for fixed throughput, when compared to Fig. 15.

Thus, the application of dynamic retransmission control provides us with a significantly improved performance which is insensitive to the population size.

6. CONCLUSION

In this paper, the dynamic behavior and stability of nonpersistent CSMA were studied using a simple linear feedback model. First, an exact analysis of this model allowed us to analytically obtain the throughput-delay performance of nonpersistent CSMA as well as the effect of the retransmission delay on the channel performance. Secondly, the stability theory based on this model (proposed in [4] in the context of slotted ALOHA) which characterizes the instability phenomenon by defining stable and unstable channels, was reviewed and shown to be applicable to CSMA. It was shown that CSMA theoretically exhibits a behavior similar to ALOHA. However, in practical situations, as long as $M \leqslant 1000$, we showed that CSMA practically provides excellent true stable performance. We also gave an analysis (based on stationary input rate) for the channel performance during the uptime of unstable channels. Since most practical systems have peak load periods followed by slack periods of low use, the throughput-delay-stability tradeoffs obtained will hold for such systems as long as the peak periods do not approach FET. Finally, we showed that the applicability of dynamic control can further improve the perforamnce significantly and can support far larger populations than can the uncontrolled case.

APPENDIX A

Solution of the System $\Pi = \Pi P$

Given the special form of matrices R and Q (see Eqs. (3) and (5)), we note that the matrix P is almost triangular, that is, for all $i \in (0, M)$,

$$p_{ij} \begin{cases} \neq 0, & (i-1)^+ \leqslant j \leqslant M - (M - i - T - 2)^+ \\ = 0, & \text{otherwise} \end{cases}$$

where for any integer α, $(\alpha)^+ \triangleq \max(0, \alpha)$, particularly $p_{ij} = 0$ for $j < i - 1$. Therefore $\Pi = \Pi P$ can be solved recursively by using the following formula

$$\pi_{j+1} = \left[\pi_j - \sum_{i=(j-T-2)^+}^{j} \pi_i p_{ij} \right] \Big/ p_{j+1,j} \tag{A.1}$$

starting with $\pi_0 = 1$. Π is then normalized so that its elements sum to one.

It is clear for the above algorithm that in computing π_{j+1} we only need $\{\pi_{(j-T-2)^+}, \cdots, \pi_j\}$ and $\{p_{ij}\}_{i=(j-T-2)^+}{}^{j}$, namely, the nonzero elements of the jth column of $\boldsymbol{P}$. In the sequel, we show that we can compute individual columns of $\boldsymbol{P}$ independently, and therefore reduce the storage requirements significantly.

For any arbitrary matrix $\boldsymbol{A}$, let $(\boldsymbol{A})_{ij}$ denote the (i, j)th element of $\boldsymbol{A}$. In computing the jth column of matrix $\boldsymbol{P}'' = \boldsymbol{R}\boldsymbol{Q}^{T+1}$, we first note that for all $m \geqslant 1$, the jth column of $\boldsymbol{Q}^m$ is computed in terms of the jth column of $\boldsymbol{Q}^{m-1}$ by

$$(\boldsymbol{Q}^m)_{ij} = [1-(M-i)\sigma](\boldsymbol{Q}^{m-1})_{ij} + (M-i)\sigma(\boldsymbol{Q}^{m-1})_{i+1,j}, \quad (j-m-1)^+ \leqslant i \leqslant j. \tag{A.2}$$

Similarly, the jth column of $\boldsymbol{P}''$ is computed as

$$(\boldsymbol{P}'')_{ij} = \left[1 - \frac{(M-i)\sigma}{1-(1-\nu)^i[1-(M-i)\sigma]}\right](\boldsymbol{Q}^{T+1})_{ij} + \frac{(M-i)\sigma}{1-(1-\nu)^i[1-(M-i)\sigma]}(\boldsymbol{Q}^{T+1})_{i+1,j}, \quad (j-T-2)^+ \leqslant i \leqslant j. \tag{A.3}$$

The computation of the jth column of $\boldsymbol{P}$ requires the jth and $(j+1)$st columns of $\boldsymbol{P}''$ for the final step

$$p_{ij} = (\boldsymbol{P}'')_{ij}[1-P_s(i)] + (\boldsymbol{P}'')_{i,j+1}P_s(i). \tag{A.4}$$

This demonstrates that we need only store two columns of $\boldsymbol{P}''$ at any time, and this amounts to $2T+6$ nonzero elements.

APPENDIX B

Computation of FET

The algorithm** below solves for the variables, $\{t_i\}_{i=0}{}^{n_c}$, in the following set of (n_c+1) linear simultaneous equations:

$$t_0 = h_0 + \sum_{j=0}^{n_c} p_{0j}t_j \tag{B.1}$$

$$t_i = h_i + \sum_{j=i-1}^{n_c} p_{ij}t_j, \qquad i = 1, 2, \cdots, n_c. \tag{B.2}$$

The Algorithm [3, 4]

1) Define

$$e_{n_c} = 1$$

$$f_{n_c} = 0.$$

2) For $i = n_c - 1, n_c - 2, \cdots, 1$ solve recursively

$$e_{i-1} = \frac{1}{p_{i,i-1}}\left[e_i - \sum_{j=i}^{n_c} p_{ij}e_j\right]$$

$$f_{i-1} = \frac{1}{p_{i,i-1}}\left[f_i - h_i - \sum_{j=i}^{n_c} p_{ij}f_j\right].$$

3) Let

$$t_{n_c} = \frac{f_0 - h_0 - \sum_{j=0}^{n_c} p_{0j}f_j}{\sum_{j=0}^{n_c} p_{0j}e_j - e_0}$$

$$t_i = e_i t_{n_c} + f_i, \qquad i = 0, 1, 2, \cdots, n_c - 1.$$

**From reference [4].

The derivation of the algorithm can be found in [4, Appendix]; it is due to the fact that $p_{ij} = 0$ for $j \leqslant i-2$. Since in this algorithm the ith row $\{p_{ij}\}_{j=i}{}^{n_c}$ is used exactly once, a considerable saving in the storage requirement is obtained by realizing, as in Appendix A, that individual rows of $\boldsymbol{P}$ can be computed independently.

The ith row of $\boldsymbol{R}\boldsymbol{Q}^m$, $m \geqslant 1$, is given in terms of the ith row of $\boldsymbol{R}\boldsymbol{Q}^{m-1}$ by

$$(\boldsymbol{R}\boldsymbol{Q}^m)_{i,j} = (\boldsymbol{R}\boldsymbol{Q}^{m-1})_{i,j-1}(M-j+1)\sigma + (\boldsymbol{R}\boldsymbol{Q}^{m-1})_{i,j} \cdot [1-(M-j)\sigma], \qquad i \leqslant j \leqslant M-(M-i-m-1)^+ \tag{B.3}$$

and finally, the ith row of $\boldsymbol{P}$ is given by

$$p_{ij} = (\boldsymbol{R}\boldsymbol{Q}^{T+1})_{i,j}[1-P_s(i)] + (\boldsymbol{R}\boldsymbol{Q}^{T+1})_{i,j+1}P_s(i), \quad (i-1)^+ \leqslant j \leqslant M-(M-i-T-2)^+. \tag{B.4}$$

For the infinite population case, the rate of generation of new packets over a slot is constant, independent of the state of the system. For such a case, it is possible to give the following closed form expression for an element p_{ij} of $\boldsymbol{P}$. Let s denote the input rate of the infinite population and let $\alpha(i)$ be defined as

$$\alpha(i) \triangleq \frac{1-e^{-s}}{1-(1-\nu)^i e^{-s}}.$$

For $i \geqslant 0$, $(i-1)^+ \leqslant j \leqslant i+T+2$, we have

$$p_{ij} = \alpha(i)[1-P_s(i)]\binom{T+1}{j-i-1}s^{j-i-1}(1-s)^{T+2-j+i} + [[1-\alpha(i)][1-P_s(i)] + \alpha(i)P_s(i)]\binom{T+1}{j-i}s^{j-i} \cdot (1-s)^{T+1-j+i} + [1-\alpha(i)]P_s(i)\binom{T+1}{j-i+1}s^{j-i+1} \cdot (1-s)^{T-j+i}. \tag{B.5}$$

APPENDIX C

Recursive Solution of the System $\Pi^{(m)} = \Pi^{(m)} P^{(m)}$

The transition matrix $P^{(m)}$ is characterized by $p_{ij}^{(m)} = 0$ for $j \leq i - 2$, with the exception of $p_{n_u,0}^{(m)} = 1$. The recursive formula (A.1) to compute $\pi_{j+1}^{(m)}$ in terms of $\{\pi_{(j-T-2)^+}^{(m)}, \cdots, \pi_j^{(m)}\}$ is still valid for $j \geq 1$; however, for $j = 0$ we have

$$\pi_1^{(m)} = \frac{(1 - p_{00}^{(m)})\pi_0^{(m)} - \pi_{n_u}^{(m)}}{p_{10}^{(m)}}. \tag{C.1}$$

Given Eqs. (C.1) and (A.1), we can express each $\pi_j^{(m)}; j = 1, 2, \cdots, n_u$, as a linear combination of $\pi_0^{(m)}$ and $\pi_{n_u}^{(m)}$

$$\pi_j^{(m)} = a_j \pi_0^{(m)} + b_j \pi_{n_u}^{(m)} \tag{C.2}$$

whereby the coefficients $\{a_j\}$ and $\{b_j\}$ are computed by the following recursive system (obtained by using Eqs. (C.1) and (A.1)):

$$a_1 = \frac{1 - p_{00}^{(m)}}{p_{10}^{(m)}} \tag{C.3}$$

$$b_1 = \frac{-1}{p_{10}^{(m)}} \tag{C.4}$$

$$a_{j+1} = \left[a_j - \sum_{i=(j-T-2)^+}^{j} a_i p_{ij}^{(m)} \right] \Big/ p_{j+1,j}^{(m)} \tag{C.5}$$

$$b_{j+1} = \left[b_j - \sum_{i=(j-T-2)^+}^{j} b_i p_{ij}^{(m)} \right] \Big/ p_{j+1,j}^{(m)}. \tag{C.6}$$

Setting $\pi_0^{(m)} = 1$ and $\pi_{n_u}^{(m)} = a_{n_u}/(1 - b_{n_u})$, we solve for $\pi_i^{(m)}$, $i = 1, 2, \cdots, n_c$. $\Pi^{(m)}$ is then normalized to yield the stationary distribution desired.

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