Two Processor Conservative Simulation Analysis

Robert E. Felderman and Leonard Kleinrock*
UCLA Computer Science Department
3732L Boelter Hall
Los Angeles, CA 90024-1596

Abstract

We present some new models and their exact analysis for the problem of two processors running a conservative distributed simulation protocol. The models show that lookahead is very useful in gaining performance, but only if the processors are well balanced in processing capacity. The models allow quantitative evaluation of the improvement in speedup attributed to null messages, as well as the degradation due to a cost for breaking deadlocks. Finally, a conservative system with "free" null messages and a small amount of lookahead is shown to outperform a Time Warp system with no cost for state saving or rollback.

2 Previous Analytical Work

There has been a great deal of work in the area of conservative simulation. The bulk of it has been in creating and empirically evaluating the performance of a particular "flavor" of conservative synchronization. Our interest is in analytical results. Wagner and Lazowska [7,8] provide techniques for bounding the speedup for simulation of queueing network models and discuss optimizations of conservative techniques. Lin, Lazowska and Baer [9] examine conservative simulation of systems without lookahead. They develop a new algorithm and provide an analytical model to estimate performance. In [10] Lin and Lazowska compare the performance of Time Warp and Chandy-Misra (conservative) simulation and derive sufficient conditions for Time Warp to outperform conservative simulation. Nicol [11,12] provides performance bounds on a new conservative algorithm. Lubachevsky [13] shows that the "bounded lag" algorithm scales efficiently as the problem size grows.

Our effort in this paper is to provide a simple model for a conservative algorithm running on two processors so as to better understand the maximum improvement that can be gained by using null messages and exploiting lookahead. We also address the cost of deadlock detection and recovery to evaluate its impact on the performance of the algorithm. Finally, we compare the conservative approach to a previously published model for Time Warp [5] and show when the conservative approach outperforms Time Warp and vice versa.

3 The Model

We now describe our model for the conservative method of synchronization. Our goal is to create a model that can easily be compared to the previous models created for Time Warp [5]. Assume we have two processes each executing on its own processor (P1, P2). We use a continuous time, discrete state model, assuming that each process/processor advances along its own virtual time (simulated time) axis visiting only the integers. Each process takes an exponential amount of time to execute an event and

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advances exactly one step forward in virtual time (along its axis) after finishing the event. After advancing, each processor will send a synchronization message to the other processor with a given probability $q_i (i = 1, 2)$. Since the synchronization is conservative, no process can perform work at virtual time $v$ until it is sure that the other processor will not send it a message time stamped with a virtual time less than or equal to $v$.

As was done in [5] we exploit the Markov process defined as the difference in virtual time (position on the axes) of the two processes, and find the probability that one processor is ahead of the other by a distance $k$. Note that $|k| \leq 1$ for unimproved conservative systems.

Here are the parameters of the model, chosen to correspond with those in [5].

$$\lambda_i = \text{rate at which processor } i \text{ executes events}$$

$$a = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

$$\bar{a} = \frac{\lambda_2}{\lambda_1 + \lambda_2} = 1 - a$$

$$q_i = \mathbb{P}[i^{th} \text{ proc. sends a msg. after advancing}]$$

$$\bar{q}_i = 1 - q_i$$

We now examine several models for two processor conservative simulation.

4 A System Without Null Messages

We first solve a model where the processors do not send null messages. We assume that when a deadlock occurs it is detected and corrected after an exponential delay with mean $d/(\lambda_1 + \lambda_2)$. If $d = 0$ then a deadlock is broken instantaneously, while $d \to \infty$ means that deadlock detection and correction takes an infinite amount of time. This system can be described by a Markov chain with the following state description.

$$(D, t_1, t_2)$$

$D = \text{Actual virtual time diff. between } P_1 \text{ and } P_2$

$t_1 = P_1^{'} \text{ s belief about the virtual time difference}$

$t_2 = P_2^{'} \text{ s belief about the virtual time difference}$

Discrepancies arise between $D$, $t_1$ and $t_2$ when the processors don't inform each other about state changes. This happens often when the processors are unlikely to send messages (small $q_i$). When a processor thinks it is ahead, it does not try to advance further. When both processors believe they are leading, we have a deadlock. The state diagram for this system is shown in Figure 1. Each state is labeled with its state description $(D, t_1, t_2)$ and an alphanumeric label for calculation of the steady-state probabilities.

If the processors start out at the same virtual time $v$ (state 0), eventually, one (say $P_1$) advances to $v + 1$ and may send a message to $P_2$ (state D). Since a conservative synchronization mechanism is being employed, $P_1$ must wait to see if $P_2$ will send it a message with virtual time $v + 1$. Its only choice is to wait until the lagging processor ($P_2$) advances, at which point that processor will "flip a coin" to decide whether to send a message. If a message is sent, $P_1$ receives it and is able to continue processing again (state 0). If a message isn't sent, $P_1$ thinks it is still ahead of the other processor and will not continue processing (state F). If $P_2$ were to advance again and not send a message, it would think (correctly) it was now ahead of $P_1$ and stop processing (state G). At this point we have a deadlock that must be broken. Deadlock detection and recovery algorithms have been discussed in [4]. Essentially, we break the deadlock by letting each processor know where it is relative to the other processor. In this example, $P_1$ would learn it was behind and begin processing, thus breaking the deadlock. If, on the other hand, each processor is able to notify the other that it has advanced its local clock, then the lagging processor is able to advance whether or not a "data" message is sent. This latter type of notification is referred to as the "null message" technique that is used to speed up conservative models. When used, we assume that this information (null messages) is propagated without cost.

The balance equations for this system can be derived from Figure 1.

$$\lambda_1 p_A = \lambda_2 q_2 p_0$$

$$\lambda_2 p_B = \lambda_1 q_1 p_0$$

$$\lambda_1 p_C = \lambda_2 q_2 p_0 + \lambda_2 q_2 p_F + \frac{\lambda_1 + \lambda_2}{d} p_G$$

$$\lambda_2 p_D = \lambda_1 q_1 p_0 + \lambda_1 q_1 p_E + \frac{\lambda_1 + \lambda_2}{d} p_H$$

$$\lambda_1 p_B = \lambda_2 q_2 p_0 + \lambda_1 q_1 p_C$$

$$\lambda_2 p_F = \lambda_1 q_1 p_A + \lambda_2 q_2 p_D$$

$$\frac{\lambda_1 + \lambda_2}{d} p_G = \lambda_2 q_2 p_F$$

$$\frac{\lambda_1 + \lambda_2}{d} p_H = \lambda_1 q_1 p_E$$

$$\frac{\lambda_1 + \lambda_2}{d} p_I = \lambda_1 q_1 p_A + \lambda_2 q_2 p_B$$

We first solve explicitly for $(p_0, p_A, p_B, p_C, p_D, p_E)$.
Figure 1: State diagram for conservative synchronization with no null messages and a cost for breaking deadlocks.

\[ p_F, p_G, p_H, p_I \].

\[
\begin{align*}
p_A &= \frac{a \rho_0 \bar{q}_2}{a} \\
p_B &= \frac{a \rho_0 \bar{q}_1}{a} \\
p_C &= \frac{\rho_0 (1 - a \bar{q}_2)}{a} \\
p_D &= \frac{\rho_0 (1 - a \bar{q}_1)}{a} \\
p_E &= \frac{\rho_0 \bar{q}_1}{a} \\
p_F &= \frac{\rho_0 \bar{q}_2}{a} \\
p_G &= \rho_0 \bar{q}_1^2 \\
p_H &= \rho_0 \bar{q}_2^2 \\
p_I &= \rho_0 \bar{q}_1 \bar{q}_2 \\
\end{align*}
\]

\[ p_0 = \frac{(a \bar{a})}{\left(1 - a \bar{a} + \bar{q}_1 + \bar{q}_2 + a \bar{a} d (3 (1 - q_1 - q_2) - q_1 q_2 + (q_1 + q_2)^2)\right)} \]

We then find the rate at which the two processors move forward in virtual time as

\[ R_2 = (\lambda_1 + \lambda_2) \rho_0 + a (p_A + p_C + p_B) + \lambda_2 (p_B + p_D + p_F) \]

We compare this rate to the equivalent single processor rate

\[ R_1 = \frac{\lambda_1 + \lambda_2}{2} \]

which is the average rate of virtual time progress if both processes are run on a single processor where additional synchronization is not necessary. Speedup is defined as the ratio of the two rates.

\[ S = \frac{R_2}{R_1} = \frac{2 (\rho_0 + a (p_A + p_C + p_B) + \bar{a} (p_B + p_D + p_F))}{4 \rho_0 (3 - q_1 - q_2)} \]

For the simple case where \( q_1 = q_2 = q \) the formula for speedup reduces to

\[ S = \frac{4 a \bar{a} (3 - 2 q)}{3 - 2 q - a \bar{a} (1 - 3 d q^2)} \]

and if the cost of breaking a deadlock is zero \( d = 0 \) then the formula reduces to

\[ S = \frac{4 a \bar{a} (3 - 2 q)}{3 - a \bar{a} - 2 q} \]

and if \( a = 1/2 \), then

\[ S = \frac{3 - 2 q}{\frac{1}{4} - 2 q} \]

We show Equation 1 plotted versus \( a \) and \( q \) for various values of \( d \) in Figure 2. We note here that the conservative system with no cost for sending messages performs better as \( q \), the interaction parameter, increases. This is in contrast to the Time Warp system [5] where speedup decreased as \( q \) increased. In the conservative system we are better off sending a large number of messages because the messages keep...
each process informed as to the virtual time progress of the other thus allowing potential parallelism to be exploited. When more messages are sent, the processors are less likely to be waiting due to lack of information and less likely to become deadlocked.

It is also clear from the figures that the cost of deadlock has a large impact on the performance if the probability of interaction is small. This is to be expected, since the probability of deadlock is higher when the processes exchange information infrequently. We can take the derivative of speedup with respect to \( d \) (the cost of breaking deadlock) to quantify the effect of \( d \) on performance.

\[
\frac{\partial S}{\partial d} = \frac{-12a^2d^2(3-2q)q^2}{(3-2q-a^2(1-3d^2))^2}
\]

We plot this function versus \( d \) and \( q \) for \( a = 1/2 \) in Figure 3 and see that changes in \( d \) have a large effect on \( S \) when \( q \) is small.

Returning again to speedup, we note that Equation 1 is only valid if \( q_1 > 0 \) and \( q_2 > 0 \). If both of these values are equal to zero (i.e., we never send messages), then speedup reduces to

\[
S = \frac{4a\bar{a}}{\bar{a}(1 + ad) + a^2}
\]

and if \( d = 0 \) in this case we get

\[
S = \frac{4a\bar{a}}{\bar{a} + a^2}
\]

Figure 2: Speedup versus \( a \) and \( q \) for various values of \( d \).

Figure 3: Derivative of speedup with respect to \( d \) (the cost of breaking a deadlock) versus \( q \) and \( d \) for \( a = 1/2 \).
5.1 Types of Lookahead

In order to formulate a model for a system using lookahead, we need to be very precise about what sort of future prediction is available. One example of this future prediction is that a process might always be able to inform the other processes of the virtual time of the next message it is going to send, but not the contents. With this sort of information, the receivers in a conservative system would be able to process all messages that had virtual times less than the time of the "scheduled" virtual time of the next message. In a two processor system each processor would execute messages with timestamps less than the virtual time of the "future" message, then wait for the arrival of that message. This system has the same performance as a TW system with no cost for state saving and rollback. TW is really forced to "wait" for the arrival of the message, but it is actually just performing useless work instead of waiting. Both systems return to processing useful work at the instant that the "straggler" message arrives.

Another type of lookahead is information that bounds the virtual time of future messages. The typical example (a FIFO queue) was given in the previous section. If we know something about the process that is being simulated, we may be able to provide information to downstream processes.

The type of lookahead that we use in our model was introduced by Nicol [11]. We can think of lookahead as the ability to transmit messages in our future to other processors. The farther into the future we are allowed to "precompute", the more lookahead we have. Nicol points out that there are two pieces of information contained in a lookahead message. The first is the virtual time of the pending message, the other is the actual contents of the message. Our previous example conveys only virtual time information while, in general, we could transmit both virtual time and data information. Nicol calls the lookahead with time and data information "full lookahead" while the time only message is "time lookahead". We use the idea of full lookahead in the next model due to its analytical tractability.

6 The Lookahead Model

Our definition of lookahead is based on our previous model that only allows processors to advance a single step in virtual time when advancing. By assuming that the processes have $K$-step full lookahead, each of the two processes is able to be at most $K + 1$ units of virtual time (events) ahead of the other (as opposed to $K$ messages ahead). Essentially we believe that a process is able to give the other process the content of any messages up to $K$ virtual time units in the
future. By assuming that null messages are used, each processor always knows its position relative to the other. Note that if \( K = 0 \), this model reverts to the simple no-lookahead model where a processor must wait when it gets ahead at all. The state diagram for this system is very simple and is shown in Figure 5. The balance equations for this system are:

\[
\lambda_1 p_k = \lambda_2 p_{k+1} \quad k = -K - 1, \ldots, 0, \ldots, K
\]

\[
p_0 = 1 - \sum_{i=1}^{K+1} (p_i + p_{i-1})
\]

The solution is

\[
p_k = \left( \frac{a}{a_0} \right)^k p_0 \quad k = -K - 1, \ldots, 0, \ldots, K + 1
\]

\[
p_0 = \frac{a^{K+1} (a - \bar{a}) \bar{a}^{K+1}}{a^{3+2K} - \bar{a}^{3+2K}}
\]

Speedup relative to the equivalent single processor implementation is

\[
S = \frac{4a\bar{a} (a^{2+2K} - \bar{a}^{2+2K})}{a^{3+2K} - \bar{a}^{3+2K}} \quad a \neq \frac{1}{2}
\]  \hspace{1cm} (6)

\[
S = \frac{4(1 + K)}{3 + 2K} \quad a = \frac{1}{2}
\]  \hspace{1cm} (7)

Equation 6 is plotted versus \( a \) and \( K \) in Figure 6. We can see from this figure that lookahead is extremely useful when the processors are nearly balanced in processing speed (\( a = 1/2 \)). In the imbalanced situation, the faster processor quickly runs out to its limit of \( K \) steps, then waits for the other processor to move forward before it can continue again. By taking the derivative of speedup with respect to \( K \), we see this result more clearly. In Figure 7 we show \( \partial S / \partial K \).

When \( K \) is small and \( a \) is near \( 1/2 \), any change in \( K \) has a major effect on speedup, though once we move away from \( a = 1/2 \) or \( K > 5 \), the impact is significantly reduced. The moral of this story is to make sure the processes progress at nearly the same rate in virtual time or lookahead will be useless.

7 Comparison to Time Warp

We now make a direct comparison between the speedup results obtained from the previously published Time Warp models and conservative models.
Figure 8: Ratio of conservative speedup (no lookahead) to "free" Time Warp speedup.

Figure 9: Ratio of conservative speedup with $K$-step lookahead to "free" Time Warp with no lookahead.

derived in the previous sections. To clearly display the tradeoffs, we compare simplified versions of each. Figure 8 shows the ratio of speedup for the conservative model using null messages but no lookahead to Time Warp with no cost for state saving and rollback [5]. It is clear that "free" Time Warp is always a winner since the ratio never exceeds one. The optimistic approach with no cost for its aggressive computation is always better.

Let us now compare free TW to the conservative model with lookahead when both systems are operating at $a = 1/2$ and when the conservative system has $K$-step lookahead. Proponents of the optimistic approach point out that their systems work well regardless of whether lookahead is exploited. Our comparison is an attempt to see how well the conservative approach exploiting lookahead fares with respect to a Time Warp system that uses no lookahead. This ratio is plotted in Figure 9 and suggests that a little lookahead combined with null messages goes a long way. For almost any value of $K$ greater than one, we see that the conservative model outperforms "free" Time Warp (ratio $> 1$). We find the threshold where the conservative approach beats TW by solving the following inequality for $K$.

$$\frac{S_{cons}}{S_{TW}} = \frac{(1 + K)(2 + \sqrt{q})}{3 + 2K} \geq 1$$  \hspace{1cm} (8)

The condition for the conservative approach to beat

Time Warp is

$$K \geq \frac{1 - \sqrt{q}}{\sqrt{q}}$$  \hspace{1cm} (9)

For $q$ (the interaction parameter) very small we need a large lookahead, but for $q > 0.1$, $K$ only needs to be 1 or 2. Figure 10 shows the areas of the $q - K$ plane where the conservative approach beats "free" Time Warp. Note that if an optimistic system with no rollback and state saving costs is afforded the same lookahead as a conservative system with no cost for null message transmissions, the optimistic approach will always perform better since it is able to aggressively compute along the critical path for free.

8 Conclusions

This paper examined some simple two processor models for the conservative synchronization method. It showed that lookahead is very useful in gaining performance, but only if the processors are well balanced in processing capacity. The models allowed quantitative evaluation of the improvement attributed to null messages, as well as the degradation due to a cost for breaking deadlocks. Finally, a conservative system with "free" null messages and a small amount of lookahead was shown to outperform a Time Warp system with no cost for state saving or rollback. However, if they both incorporate lookahead, then TW is the winner. Unfortunately for the conservative approach, lookahead is not often easy to
Figure 10: Area of the $q - K$ plane where the conservative approach with lookahead wins out.

...come by [15,14]. A simple FIFO queueing system provides great lookahead, but add in preemptive-priority queueing and all the lookahead disappears. It may be unwise to utilize a synchronization mechanism that needs lookahead to perform well.

**References**


