ON STABILITY OF PACKET SWITCHING IN A RANDOM

MULTI-ACCESS BROADCAST CHANNEL

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Abstract

The dynamic behavior and stability of packet switching in a random multi-access broadcast data communication channel is considered. We give quantitative estimates for the relative stability of these channels and discuss the tradeoff among channel stability, delay and throughput. These considerations indicate the need for channel control procedures.

1. INTRODUCTION

Random multi-access broadcast channels have been studied in the past [1,2,3,4,5]. (Such channels are also referred to as ALOHA channels.) These previous studies obtained steady state performance results for throughput and/or delay under the assumption of equilibrium conditions. Often this assumption is not satisfied,in which case the aforementioned performance applies only for a (possibly small) interval of time. In this paper we investigate the effect of this phenomenon.

2. THE MODEL

2.1 MODEL DESCRIPTION

The broadcast channel is assumed to support a large number (N_m) of active terminals. Each terminal has buffer space for exactly one (fixed length) message packet. All terminals are assumed to synchronize their packet transmissions into fixed channel time slots. Throughout this paper, time is expressed in units of channel slots. Only when the terminal buffer is empty, may a new message packet be generated (by the terminal's external source) and this will occur with probability σ in a slot. When simultaneous independent packet transmissions are attempted by more than one terminal, these packets "destroy" each other and must be retransmitted at a later time [4]. Such packets are said to be backlogged. We let N^t be a random variable representing the total number of backlogged packets in the channel and S^t be the combined input rate of packets into all terminals at time t. The vector (N^t,S^t) is defined to be the channel state vector or channel load at time t. For $N^t = n$, $(N^t,S^t) = (n,(N_m - n)\sigma)$. Note that the channel input rate S^t decreases linearly as N^t increases.

In slotted satellite channels (see [4]) packets which "collide" at the channel are retransmitted (after a round-trip propagation delay of R slots) during one of the next K slots, each such being chosen at random with probability 1/K. Thus retransmission takes place on the average R + (K+1)/2 slots after the previous transmission. This retransmission scheme is difficult to analyze and so we assume the simpler scheme** whereby every backlogged packet independently retransmits with probability p. This is an excellent approximation for the slotted satellite channel above as shown by simulations for moderate to large K when we choose

$$p = \frac{1}{R + (K + 1)/2} \tag{1}$$

As in [4] we assume R = 12 and we express our numerical results in terms of K (using Eq. (1)) rather than p.

2.2 CHANNEL THROUGHPUT

We define S_{out} to be the delivered output (throughput) rate of the channel, which is the probability of exactly one (successful) packet transmission in a channel slot. For the model above, if $(N^t,S^t) = (n,(N_m - n)\sigma)$ then

$$S_{out} = (1 - p)^n (N_m - n)\sigma(1 - \sigma)^{N_m-n-1} + np(1 - p)^{n-1}(1 - \sigma)^{N_m-n} \tag{2}$$

In the limit when $N_m \to \infty$ and $\sigma \to 0$ (such that $N_m\sigma = S < \infty$) we have the infinite population model in which new packets are generated for transmission over the broadcast channel at the constant Poisson rate S. In this case Eq. (2) reduces to

$$S_{out} = (1 - p)^n Se^{-S} + np(1 - p)^{n-1}e^{-S} \tag{3}$$

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**This is a realizable scheme in radio channels with negligible propagation delays.

This expression is very accurate even for finite N_m if $\sigma << 1$ and if we replace $S = N_m\sigma$ by $S = (N_m - n)\sigma$.

In Fig. 1, for a fixed K we show the behavior of S_{out} as a function of the channel load (n,S) as expressed in Eq. (3). Note that there is an equilibrium contour in the (n,S) "phase plane" on which the channel input rate S is equal to the channel throughput rate S_{out}. In the shaded region enclosed by the equilibrium contour, S_{out} exceeds S; elsewhere $S > S_{out}$ (the system capacity is exceeded!). The area of the shaded region may be increased by increasing K as shown in Fig. 2 where a family of equilibrium contours are displayed.

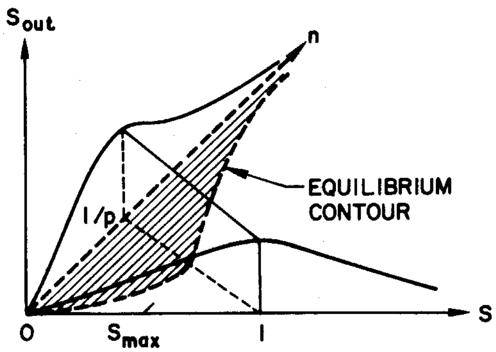

Fig. 1 Channel Throughput Rate as a Function of Load

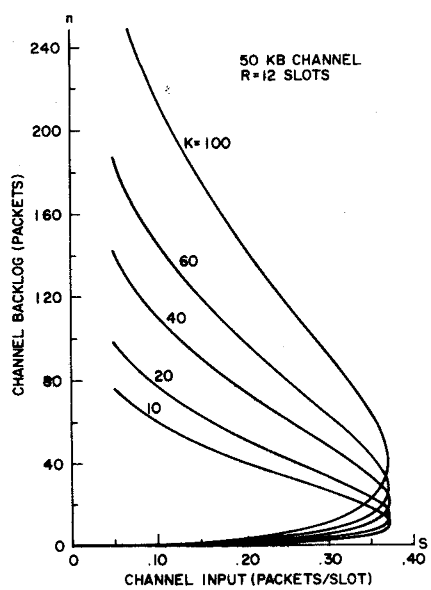

Fig. 2 Equilibrium Contours

3. TIME VARYING INPUTS

Consider the case in which $N_m = N_m(t)$ as for example shown in Fig. 3. We use the fluid approximation [6] for the trajectory of the channel state vector (N^t,S^t) on the (n,S) plane as sketched in Fig. 4; we show two possible cases corresponding to Fig. 3 for different values of N_3. (The arrows indicate the directions of "fluid" flow.) The solid line (case 1) represents a trajectory which returns to the original equilibrium point on contour C_1 despite the input pulse. The dashed line (case 2) shows a less fortunate situation in which the decrease in the channel input at time t_2 is not sufficient to bring the trajectory back into the "safe" region. Eventually the channel will be paralyzed as a result of an increasing backlog and a vanishing channel throughput rate.

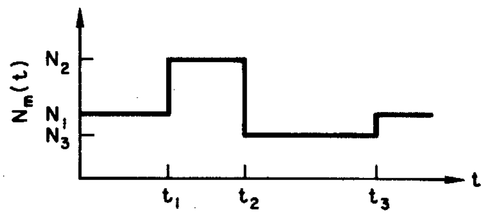

Fig. 3 $N_m(t)$

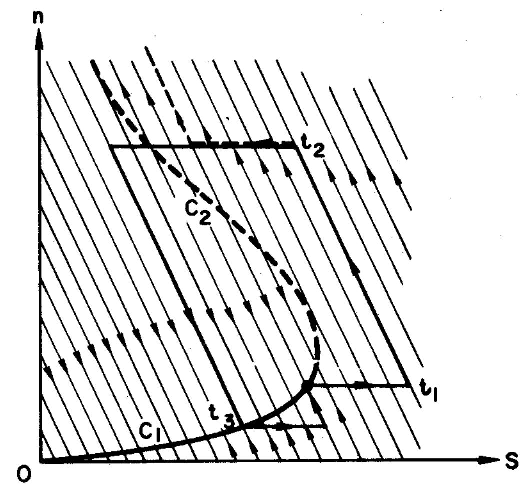

Fig. 4 Fluid Approximation Trajectories

We have demonstrated channel "instability" due to a time varying input. Next we study the conditions under which the channel with a stationary input is "unstable."

4. STATIONARY INPUTS

4.1 CHANNEL STABILITY

When both N_m and σ are constant in time we have a "stationary" input. For this case we define the <u>channel load line</u> in the (n,S) plane as the line $S = (N_m - n)\sigma$. <u>A channel is said to be stable when its load line intersects (nontangentially) the equilibrium contour in exactly one place.</u>

4.2 STABLE AND UNSTABLE CHANNELS

In Fig. 5(a) we show an example of a <u>stable</u> channel and its <u>operating point</u>. If N_m is <u>finite</u>, a stable channel can always be achieved by using a sufficiently large K (see Fig. 2). Of course, a large K implies large average packet delays which may not be desirable [4].

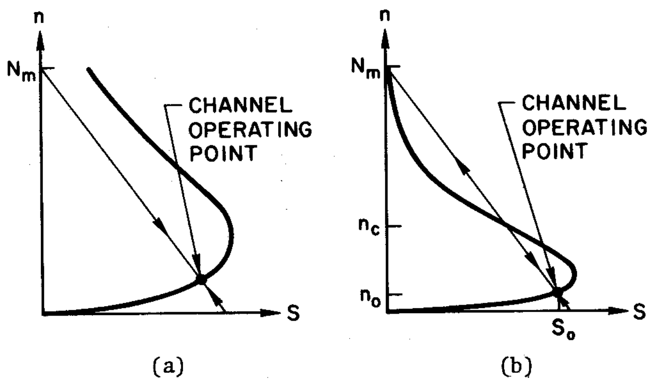

Fig. 5 Stable and Unstable Channels

In Fig. 5(b) we show an example of an unstable channel. (Note that a load line which misses or is only tangential to the equilibrium contour is also unstable by our definition.) The point (n_o, S_o) is the desired operating point since it yields the largest channel throughput and smallest packet delay. The channel, however, cannot maintain equilibrium at this operating point indefinitely since N^t is a random process; that is, with probability one, the backlog N^t crosses the "critical" value n_c in a finite time and as soon as it does S exceeds S_{out}. Under this condition, although there is a small probability that N^t may return below n_c, all our simulations showed that the channel state vector (N^t, S^t) accelerated up the channel load line producing an increasing backlog and a vanishing throughput rate. In this state, the channel was disabled and external intervention was necessary to restore proper channel operation.

4.2 A STABILITY MEASURE

From the above discussion and referring to Fig. 5(b), we divide the channel load line into two regions: the safe region consisting of the channel states $\{(n,S) | n \le n_c\}$ and the unsafe region consisting of the channel states $\{(n,S) | n > n_c\}$. A good stability measure (for these unstable channels!) is the average time to exit into the unsafe region starting from a safe channel state. To be exact we define FET to be the average first exit time into the unsafe region starting from an initially empty channel (zero backlog size). FET will be used as our measure of channel stability. Its derivation and an efficient computational procedure are given in [7].

5. NUMERICAL RESULTS

As in [4], all our numerical computations assume a 50KBPS satellite channel with 1125 bit packets and a round-trip propagation delay of 0.27 second (giving R = 12 and there are 44.4 slots in a second).

5.1 AVERAGE FIRST EXIT TIMES (FET)

In Fig. 6 we have shown FET as a function of K for the infinite population model and for fixed values of the channel throughput rate S_o at the channel operating point. The infinite population model results give us the worst case estimates, as shown in Fig. 7. In Fig. 7, we show FET as a function of N_m for K = 10 and four values of S_o. The channel FET increases as N_m decreases and there is a critical N_m below which the channel is always stable in the sense of Fig. 5(a).

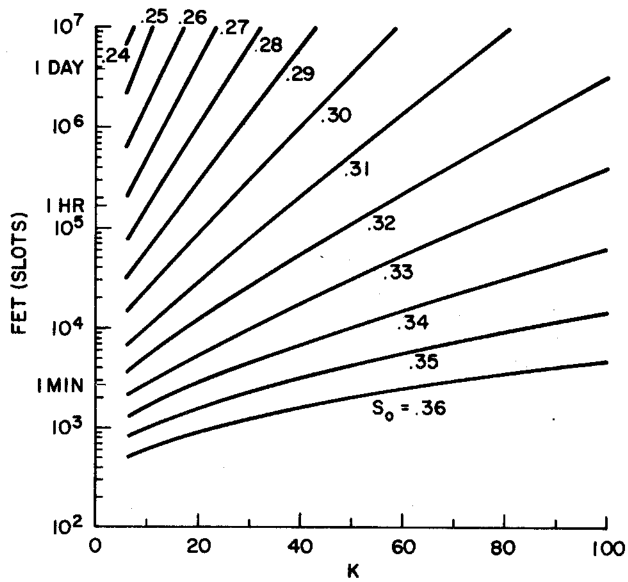

Fig. 6 FET Values for the Infinite Population Model

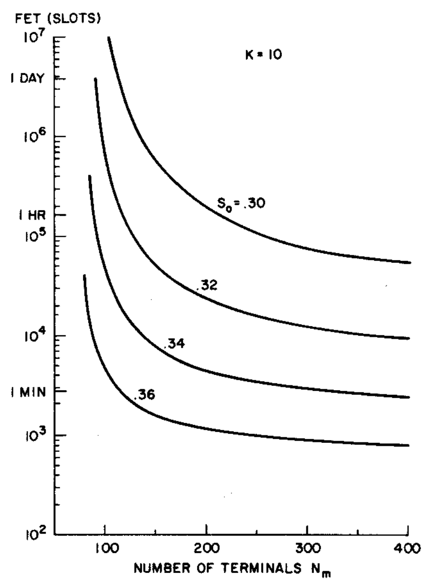

Fig. 7 FET as a Function of N_m

In Fig. 6 we see that the channel stability (FET value) can be improved either by decreasing the channel throughput rate or by increasing K (which in turn increases the average packet delay). For example, if we limit S_0 below 0.25 and using $K = 10$ the channel enters the unsafe region only once every 2 days on the average.

5.2 STABILITY-THROUGHPUT-DELAY TRADEOFF

In Fig. 8 we show two sets of throughput-delay performance curves with guaranteed FET values. (For comparison we have also shown as a lower bound the optimum performance curve which was obtained in [4] without regard to channel stability.) The first set consists of three solid curves corresponding to an infinite population model with channel FET ≥ 1 day, 1 hour, or 1 minute. The second set consists of two dashed curves corresponding to a finite population of 150 terminals with the channel FET ≥ 1 day or 1 hour. This figure displays the tradeoff among channel stability, throughput, and delay.

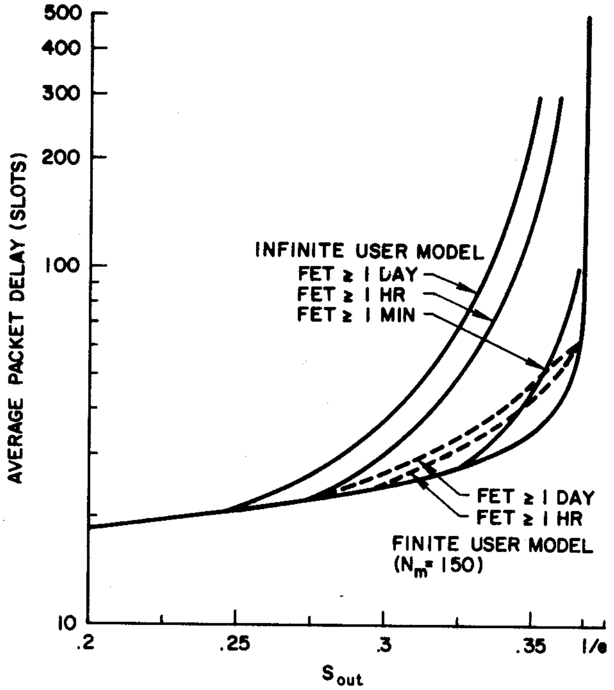

Fig. 8 Stability-Throughput-Delay Tradeoff

6. CONCLUSION

We have examined stability conditions for a packet switched random multi-access broadcast channel supporting a large number of terminals. For an unstable channel, any throughput-delay performance results obtained under steady state assumptions [1,2,3,4,5] will be achievable only for finite periods of time. This observation has been quantified here by the definition of FET as a measure of channel stability. Strategies are currently being studied to control this phenomenon. Preliminary results [7] show that the channel may be dynamically controlled in order to achieve truly stable throughput-delay performance close to the optimum.

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