## **Reviews of Books and Papers in the Computer Field**

DONALD L. EPLEY, Reviews Editor

D. W. FIFE, A. I. RUBIN, R. A. SHORT, H. S. STONE

Assistant Reviews Editors

Please address your comments and suggestions to the Reviews Editor: Donald L. Epley, Department of Electrical Engineering, University of Iowa, Iowa City, Iowa 52240

## A. SIMULATION

**R69-5** The Statistics of Discrete-Event Simulation—G. S. Fishman and P. J. Kiviat (*Simulation*, vol. 10, pp. 185–195, April 1968).

This paper discusses the problems of simulating systems in which some random behavior exists. The title refers to "discrete-event simulation," but this remains undefined and unemphasized throughout most of the paper. (The expression refers to systems in which events, such as customer arrivals, occur at a countable set of points on the continuous time axis-this as opposed to a continuous-event simulation in which important properties of the system must be accounted for over a continuum in a given time interval.) The authors trace the elements of a typical experiment and identify the statistical problems along with a discussion of their possible solutions. They isolate a number of frequently overlooked considerations in the random input process, particularly the independence of adjacent samples. Similarly, in estimating system performance, the problem of correlated outputs must be solved. In short, in any simulation, the input data and the system structure must be understood and examined carefully.

Random number generators are discussed (without defining some of the quantities used, unfortunately), and the important conclusion is referred to, namely, that currently there is no better way for generating pseudorandom numbers than that of a simple multiplicative congruence. The authors correctly state that it is important for any simulation to verify the independence of its input samples, but they neglect to tell us if the currently popular simulation languages (e.g., GPSS, SIMSCRIPT, SIMULA) provide for this.

The use of analytical models and/or simulation of subsystems is recommended as significant tools in reducing the complexity of large systems simulations. An example of a simple queueing system is considered and the question of simulation detail is discussed. Validation of the generated system model is one of the most critical considerations in simulations. This can be accomplished only if there is available some numerical data from the actual system. The chi-squared test is referred to and its shortcoming for small sample size is pointed out; the variance test is recommended as an excellent substitute in this case. It is suggested that spectrum estimation be used as a means for determining independence of output samples; they fail to point out that the periodogram so obtained will not converge to the true spectrum unless aliasing is used (i.e., the variance of the estimate does not behave properly). Choice of sampling interval is discussed, but should have been elaborated upon (it is here that they make the significant point that process activity is the critical driving influence in the simulation). Output variance reduction methods using the

powerful technique of antithetic random variables is mentioned. The authors close on the point that little work has been accomplished in developing useful methods for deciding where in the parameter space one should collect samples. This is a deep and difficult problem.

As the authors promised, they have provided in this paper a description of the problem areas and their significance, with few solutions to these problems. This reviewer feels that they have succeeded in this endeavor and would recommend the paper and many of its references to anyone seriously interested in simulation methods.

LEONARD KLEINROCK Dept. of Engrg. University of California Los Angeles, Calif.

## **B. HYBRID COMPUTATION**

**R69-6** Trajectory Optimization by a Direct Descent Process— L. E. Fogarty and R. M. Howe (*Simulation*, vol. 11, pp. 145–155, September 1968).

This paper develops and investigates a practical method by which certain aerospace trajectory optimization problems with terminal constraints can be relatively easily solved on a hybrid computer. If a gradient technique has been selected for the optimization process, then it has generally been the practice to solve the equations adjoint to the equations of motion by utilizing a digital computer. In the present work no utilization is made of the adjoint variables. The gradient is calculated by determining the response of the cost function to approximately impulsive control perturbations, similar to the method used by Wingrove and Raby. The variational problems which are considered are those of the Mayer type where the cost function to be minimized is a function of the terminal state and terminal time. In this paper a technique is developed so that the gradient which is calculated is compatible with imposed terminal constraints. The calculation of this modified gradient, which consists of a linear combination of the cost gradient and the gradients of the terminal constraints, is relatively simple and can be rapidly performed on the hybrid computer. The circuitry required for handling the terminal constraints by this technique, as well as that required to simulate the other equations of the problem, is described in substantial detail.

To illustrate the applicability of the method, two example problems are presented. The first is the classical brachistochrone prob-