Approximate Output Processes in Hidden-User Packet Radio Systems

HIDEAKI TAKAGI, MEMBER, IEEE, AND, LEONARD KLEINROCK, FELLOW, IEEE

Abstract—**The processes consisting of the packet interdeparture times for contention-type packet broadcasting systems in a hidden-user, single-hop environment are studied under the heavy-traffic assumption. The channel access protocols considered include pure ALOHA and unslotted nonpersistent carrier-sense-multiple-access (CSMA). The theory of superposition of independent renewal processes is applied to approximate the distribution of the duration of each unsuccessful transmission period in channel state. Our analysis results for the channel throughput and the coefficient of variation for the packet interdeparture time in symmetric configurations are shown to be in good agreement with simulation results over a wide range of offered channel traffic.**

I. Introduction

In [12], we conducted an exact stochastic analysis of packet interdeparture times (i.e., intervals between two consecutive successful transmissions) for several channel access protocols in packet broadcasting systems. Channel access protocols which we called *memoryless protocols*, such as slotted ALOHA [9], pure ALOHA [1], and slotted and unslotted nonpersistent carrier-sense-multiple-access (CSMA) [7], [13], were studied to find explicitly the distributions of packet interdeparture times. (The reciprocal of the mean interdeparture time is the channel throughput, one of our main performance measures.) In our earlier treatment of CSMA, it was assumed that every user is in line of sight of all others so that any transmission can be heard (after a finite signal propagation delay) by all parties (i.e., a *fully-connected* configuration). Regulation of transmission by listening to other transmissions is the essense of CSMA, and it is that which achieves a high throughput (as long as the propagation delay is small compared to the packet transmission time).

However, in applying CSMA to ground-based packet radio communication systems for a population of geographically distributed users, such as PRNET [6], there are many situations in which some users cannot hear transmissions from certain other users; this is possibly because they are out of transmission range of each other or because they are separated by some physical obstacles (e.g., mountains) blocking the signal. Such a situation, called the *hidden-terminal problem,* was analyzed by Tobagi and Kleinrock [13], [14] and a serious throughput degradation was shown to exist. This is because hidden users behave independently, ignoring the ongoing transmissions. (The busy-tone multiple-access (BTMA) was then proposed to save the day.)

This paper focuses on the performance analysis of hidden-user configurations by use of an approach different from [14]. Our method is based on the modeling of packet transmission activity at each user as a two-state (transmitting or not) alternating renewal process. In our models, we assume that each user has packets ready for transmission at all times (*heavy-traffic* assumption). Also, the transmission protocol is assumed to be memoryless in the sense that whenever a user experiences an idle (nontransmitting) period, he renews his action regardless of the past happenings. Now, let us define the two alternating states in channel. The *transmission state* in channel is the state where at least one user is transmitting or any transmission is being sensed. Also, the channel *idle state* is defined as the state where no users are transmitting or no transmissions are being sensed. Thus, the channel state also alternates between the transmission and idle periods. (There can be two consecutive transmission periods with an idle period of duration 0 between them.)

A transmission period is *successful* if there are no other transmissions heard at the intended receiver during the (protocol-dependent) vulnerable period. Exact analysis is possible for the stochastic property of the durations of the channel idle period and a successful transmission period. However, to analyze the duration of an *unsuccessful* transmission period, an approximation using the theory of superposition of independent renewal processes is applied. This treatment involves a twofold approximation: i) we treat each user's transmission process as independent with a properly reduced transmission rate (whereas, in fact, two CSMA users in line of sight of each other behave dependently); ii) we treat consecutive interevent times in a superposed process as if they were independent and identically distributed (whereas in reality they are not). The validation of our approximation will be provided by comparing the results with simulation.

We assume the existence of a single receiving station which is in line of sight of all users (*single-hop* systems). We now realize a spectrum of *hiddenness* ranging from ALOHA (completely hidden) to fully-connected CSMA (completely visible) and the partially hidden configuration of CSMA in between. Our approach makes it possible for the solution to range smoothly over all degrees of hiddenness, as opposed to the one in [14] where, for example, the expression for the channel throughput in the limit of fully-connected CSMA (in a zero propagation delay, infinite population model) does not agree with the exact result in [7]. Also, through (approximate) analysis of the durations of alternating channel states (idle and transmitting), we obtain an approximation to the mean and variance of the packet interdeparture time. These are used to determine the coefficients in the diffusion process approximation to users' packet queue length distribution in [11].

This paper is organized as follows. In Section II, we show a brief extract from [12] of the analysis of packet output processes in contention-type memoryless systems. In Section III, we quote some results on the superposition of renewal processes. Based on these preliminaries, we analyze the packet departure processes of pure ALOHA (Section IV) and unslotted nonpersistent CSMA (Section V) systems. Comparison of our calculated results with the simulation results in

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H. Takagi is with the Department of Computer Science, University of California, Los Angeles, CA 90024, and the IBM Japan Science Institute, Tokyo, Japan.

L. Kleinrock is with the Department of Computer Science, University of California, Los Angeles, CA 90024.

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some example systems is discussed in Section VI. We also discuss the rationale of our assumptions and give directions to possible refinement of the present formulation. Our presentation in this paper is restricted to unslotted systems consisting of statistically identical users. See [10] for a similar treatment of slotted CSMA systems and cases of nonidentical users.

Throughout the paper, the packet length is assumed to be constant, and its transmission time is chosen as the unit of time. Then, in typical ground-based systems, the signal propagation delay, denoted by a, is small (e.g., $a = 0.01$ normalized time units). In the analysis of pure ALOHA (Section IV) we assume that $a = 0$. We consider the case $a \geqslant 0$ for CSMA systems (Section V) with various degrees of hiddenness. The completely hidden case reduces to pure ALOHA with $a \geqslant 0$.

II. Analysis of Output Processes

As shown in [12], in contention-type systems with heavy traffic, a packet interdeparture time X consists of $K - 1$ cycles of alternating channel idle periods $\{I^{(k)}\}$ and unsuccessful transmission periods $\{F^{(k)}\}$ $(k = 1, 2, \cdots, K - 1)$ terminated by the last cycle of $I^{(K)}$ and a successful transmission period T:

$$X=\sum_{k=1}^{K-1}[I^{(k)}+F^{(k)}]+I^{(K)}+T. \tag{1}$$

In particular, for memoryless protocols $\{I^{(k)} + F^{(k)}; k = 1, 2, \cdots, K - 1\}$ and $\{I^{(K)} + T\}$ are mutually independent and also each of them is independent of K. Thus, we can express the mean and variance of X as

$$\bar{X}=(\bar{K}-1)(\bar{I}+\bar{F})+\bar{I}+\bar{T} \tag{2}$$

$$\text{Var }[X]=\bar{K}\text{ Var }[I]+(\bar{K}-1)\text{ Var }[F]+\text{Var }[T]+(\bar{I}+\bar{F})^2\text{ Var }[K] \tag{3}$$

where I and F represent each of $\{I^{(k)}\}$ and $\{F^{(k)}\}$ identically distributed, respectively, and we have assumed the independence of $I^{(k)}$, $F^{(k)}$, and T for each k. For contention-type, memoryless protocols, K is geometrically distributed as

$$\text{Prob }[K=k]=(1-\gamma)^{k-1}\gamma \qquad k=1,2,\cdots$$

$$\bar{K}=\frac{1}{\gamma};\quad \text{Var }[K]=\frac{1-\gamma}{\gamma^2} \tag{4}$$

where γ is the probability that a transmission is successful once it has been started by breaking the channel idle period.

The throughput S and the coefficient of variation C^2 of packet interdeparture times for the whole system are given by

$$S=\frac{1}{\bar{X}};\ C^2=\frac{\text{Var }[X]}{\bar{X}^2}. \tag{5}$$

For fully-connected CSMA systems and an infinite population of pure ALOHA users, we have found the exact expressions for the distribution of X as well as for the mean and variance of X in [12]. For systems involving a finite number of hidden users, however, it seems very difficult to find the distribution of F for the reasons mentioned in Section I. Therefore, we introduce several approximations based on the analysis of superposed renewal processes.

III. Superposition of Independent Renewal Processes

We assume that there are a finite number M of event sources (indexed 1 through M), at each of which events occur from time to time independently of the others. Let the interevent times at source i be independent and identically distributed as represented by Y with mean $\bar{Y}$ and distribution function $F(x)$, $x \geqslant 0$ (identical for $i = 1, 2, \cdots, M$). Fig. 1 illustrates a combination of these events into a superposition process. Note

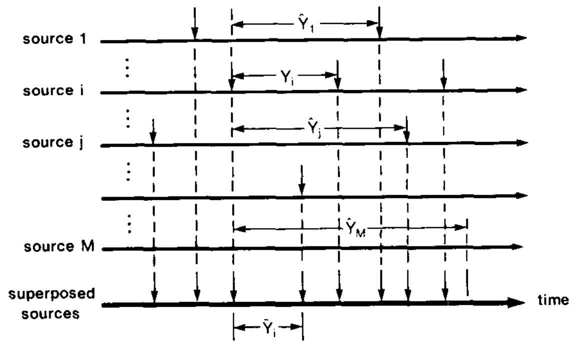


Fig. 1. Interevent time in the superposition process.

that in the superposition process, the interevent times are generally neither independent nor identically distributed. (The correlation among such successive intervals was studied by Lawrance [8] and Ito [5].) However, we use here only the steady-state distribution of a single interevent time, denoted by $\tilde{Y}$, following an arbitrarily chosen event.

Conditioned on picking an event from source i, the following interevent time $\tilde{Y}(i)$ can be expressed as

$$\tilde{Y}(i)=\min\,[Y_i,\ \hat{Y}_1,\ \cdots,\ \hat{Y}_{i-1},\ \hat{Y}_{i+1},\ \cdots,\ \hat{Y}_M] \tag{6}$$

where $\hat{Y}_j$ stands for the residual life in source j whose pdf is given by $[1 - F(x)]/\bar{Y}$. Thus, the distribution of $\tilde{Y}$ is given by [2]

$$\text{Prob }[\tilde{Y}>x]=[1-F(x)]\cdot\left\{\frac{1}{\bar{Y}}\int_x^\infty[1-F(y)]\,dy\right\}^{M-1} \qquad x\geqslant 0. \tag{7}$$

As an example, assume that

$$F(x)=\begin{cases}0 & 0\leqslant x<1\\ 1-e^{-g(x-1)} & x\geqslant 1\end{cases}$$

$$\bar{Y}=1+\frac{1}{g} \tag{8}$$

which is an exponential distribution shifted by 1. This example will be used later in the distribution of the random variable $\tilde{Y}$ when $\tilde{Y} \leqslant 1$ (we denote this random variable by f).

$$\text{Prob }[f\leqslant x]\triangleq\text{Prob }[\tilde{Y}\leqslant x\,|\,\tilde{Y}\leqslant 1]=\frac{1-\text{Prob }[\tilde{Y}>x]}{1-\text{Prob }[\tilde{Y}>1]} \tag{9}$$

or

$$\text{Prob }[f>x]=\frac{[1+g(1-x)]^{M-1}-1}{(1+g)^{M-1}-1} \qquad 0\leqslant x\leqslant 1 \tag{10}$$

from which we can calculate

$$\bar{f}=\frac{1}{(1+g)^{M-1}-1}\left[\frac{(1+g)^M-1}{gM}-1\right]$$

$$\text{Var }[f]=\frac{1}{[(1+g)^{M-1}-1]^2}\cdot\left\{\frac{2[(1+g)^{M+1}-1][(1+g)^{M-1}-1]}{g^2M(M+1)}-\frac{[(1+g)^M-1]^2}{(gM)^2}-(1+g)^{M-1}+\frac{2(1+g)^{M-1}}{M}\right\}. \tag{11}$$

We note that the limiting forms of the expressions in (10) and (11) for $M \to \infty$ with G fixed at $G = gM$ give

$$\text{Prob } [f \leqslant x] = \frac{1-e^{-Gx}}{1-e^{-G}} \qquad 0 \leqslant x \leqslant 1$$

$$\bar{f} = \frac{1}{G} - \frac{e^{-G}}{1-e^{-G}}; \quad \text{Var } [f] = \frac{1}{G^2} - \frac{e^{-G}}{(1-e^{-G})^2} \tag{12}$$

which can also be obtained by considering a collective Poisson stream with rate G [12].

IV. Output Processes of Pure ALOHA Systems

We first consider output processes with pure (or unslotted) ALOHA for a finite number M of users. We assume that the propagation delay $a = 0$. (The case of nonzero a is given as a special case of the system analyzed in Section V.) Whether hidden or not, each user of pure ALOHA behaves independently of all others. So, let each user alternate between the transmitting state of duration 1 and the idle state of duration exponentially distributed with mean $1/g$. Thus, if we focus attention on the instants of starting transmission at each user, the intervals between those instants are independent and identically distributed as given by the distribution function in (8). The interval between two arbitrarily chosen successive starts of transmissions in the whole system is distributed as given by (7).

Now, let us give γ and the distributions of I, F, and T for pure ALOHA. First, obviously the channel idle period is exponentially distributed with aggregate parameter gM:

$$\text{Prob } [I \leqslant y] = 1 - e^{-Mgy} \qquad y \geqslant 0$$

$$\bar{I} = (gM)^{-1}; \quad \text{Var } [I] = (gM)^{-2}. \tag{13}$$

A successful transmission is obtained when a packet which breaks the channel idle period is not overlapped by any other transmission during its entire transmission period of length 1. Thus,

$$\gamma = e^{-(M-1)g}. \tag{14}$$

The duration of a successful transmission period is constant:

$$T = 1. \tag{15}$$

The results in (13)–(15) are exact. It remains for us to find the distribution of F. Note that F consists of an indefinite number of successive transmissions such that the intervals between their successive start times are all less than 1. Such an interval when arbitrarily chosen is distributed as in (9). A difficulty arises in finding the distribution of F; the successive intervals between transmission start times are neither independent nor identically distributed. However, let us introduce an approximation that they are independent and identically distributed as is given by (9). Thus, defining $f^{(n)}$ as the nth such interval in an interval F, we have

$$F = \sum_{n=1}^{L} f^{(n)} + 1. \tag{16}$$

The number of transmissions contained in an unsuccessful transmission period, denoted by L, is geometrically distributed (by approximation) as

$$\text{Prob } [L = n] \cong (1-\delta)^{n-1}\delta \qquad n = 1, 2, \cdots$$

$$\bar{L} = \frac{1}{\delta}; \quad \text{Var } [L] = \frac{1-\delta}{\delta^2} \tag{17}$$

where δ is the probability that an arbitrary interval between two successive transmission start times is no shorter than the packet transmission time 1. In the context of a superposition process, it is given by

$$\delta \triangleq \text{Prob } [\tilde{Y} \geqslant 1] = (1+g)^{-(M-1)}. \tag{18}$$

We are now in a position to calculate $\bar{F}$ and Var $[F]$ by use of the formula for the sum of independent random variables. From (16), they are expressed as

$$\bar{F} = \bar{L}\bar{f} + 1; \quad \text{Var } [F] = \bar{L} \text{ Var } [f] + \bar{f}^2 \text{ Var } [L] \tag{19}$$

where f is the generic representation of the identically distributed $f^{(n)}$'s, and its distribution is given by (10). Also $\bar{L}$ and Var $[L]$ are given by (17) and (18). Thus, we get

$$\bar{F} = \frac{(1+g)^M - 1 - gM(1+g)^{-(M-1)}}{gM[1-(1+g)^{-(M-1)}]} \tag{20}$$

$$\begin{aligned}\text{Var } [F] = & \frac{2(1+g)^{M-1}[(1+g)^{M+1}-1]}{g^2M(M+1)[(1+g)^{M-1}-1]} \\ & + \frac{(1+g)^{M-1}[(1+g)^M-1]^2[(1+g)^{M-1}-2]}{g^2M^2[(1+g)^{M-1}-1]^2} \\ & + \frac{(1+g)^{M-1}}{[(1+g)^{M-1}-1]^2}\left\{\frac{2(1+g)^{M-1}}{M}\right. \\ & \left. - \frac{2[(1+g)^M-1][(1+g)^{M-1}-1]}{gM} - 1\right\}.\end{aligned} \tag{21}$$

It is interesting to examine the validity of our approximation just introduced by comparing $\bar{F}$ given in (20) and the exact expression for $\bar{F}$ (obtained in the Appendix):

$$\bar{F}_{\text{exact}} = \frac{(1+g)^M - 1 - gMe^{-g(M-1)}}{gM[1-e^{-g(M-1)}]}. \tag{22}$$

The difference between (20) and (22) only exists between the terms $(1 + g)^{-(M-1)}$ and $e^{-g(M-1)}$ which are close when M is large with G fixed at $G = gM$ and are identical in the limit $M \to \infty$.

Thus, we have expressed all variables needed to evaluate (5) in terms of g and M. The numerical results in some example configurations are provided later along with those for unslotted CSMA (see Fig. 2).

V. Output Processes of Unslotted CSMA Systems

We now proceed to study the packet interdeparture times for a population of unslotted nonpersistent CSMA users in a hidden-user environment with a fixed propagation delay $a \geqslant 0$. We assume a symmetric hidden-user configuration which consists of M identical users, each of whom can hear transmissions from m users (including himself). So, the case $m = 1$ corresponds to a pure ALOHA system while the case $m = M$ is equivalent to a population of fully-connected CSMA users.

Given the rate of starting transmission g by each user, the distribution of a channel idle period I is given in (13). The probability that a user who initiates a transmission period gets a success is then given by

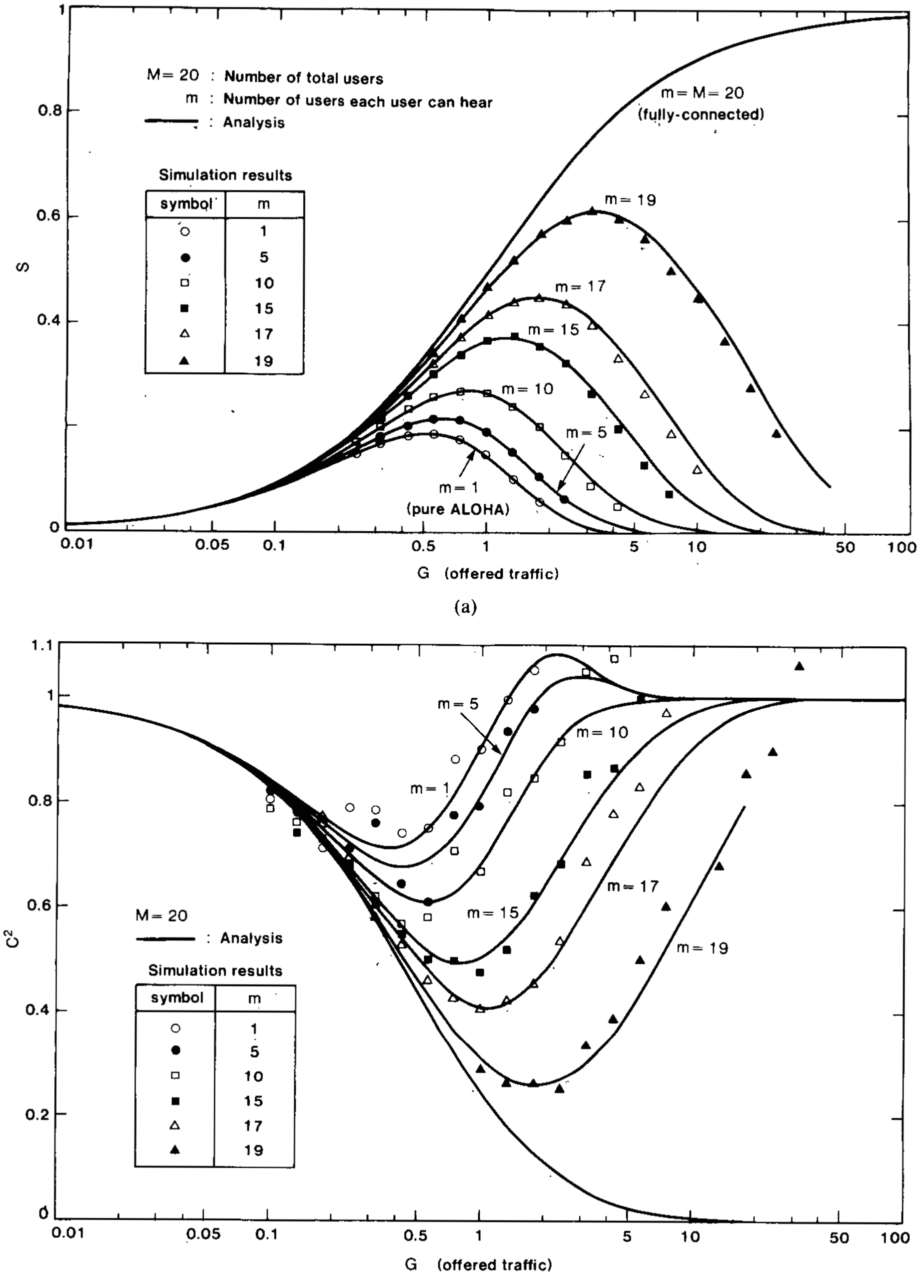
$$\gamma = \gamma_1 \cdot \gamma_2 \tag{23}$$

where

$$\gamma_1 = \exp[-(1+a) \cdot g(M-m)];$$

$$\gamma_2 = \exp[-a \cdot g(m-1)]. \tag{24}$$

It should be clear that γ_1 accounts for the probability that



(b)

Fig. 2. (a) Throughput (S) and (b) coefficient of variation of the packet interdeparture time (C^2) in unslotted CSMA (zero propagation delay) for symmetric hidden-user configurations.

users hidden from a user who initiates the transmission period do not start transmission during time $1 + a$, and that γ_2 is the probability that those users who can hear the leading transmission do not start transmission during time a. The duration of a successful transmission period is constant:

$$T = 1 + a. \tag{25}$$

In order to deal with the duration of an unsuccessful transmission period F, we distinguish two kinds of unsuccessful transmission periods; the first kind (whose duration is denoted by $F^{(1)}$) is one such that no transmissions by the users hidden from the initiating user are involved in the transmission period, and the second kind (whose duration is denoted by $F^{(2)}$) is one containing transmissions from hidden users. Thus, we have

$$F = \begin{cases} F^{(1)} & \text{with probability } \dfrac{\gamma_1(1-\gamma_2)}{1-\gamma} = \dfrac{\gamma_1 - \gamma}{1-\gamma} \\ F^{(2)} & \text{with probability } \dfrac{1-\gamma_1}{1-\gamma} \end{cases} \tag{26}$$

and so

$$\bar{F} = \frac{\gamma_1 - \gamma}{1-\gamma} E[F^{(1)}] + \frac{1-\gamma_1}{1-\gamma} E[F^{(2)}]$$

$$\text{Var } [F] = \frac{\gamma_1 - \gamma}{1-\gamma} E[\{F^{(1)}\}^2] + \frac{1-\gamma_1}{1-\gamma} E[\{F^{(2)}\}^2] - \bar{F}^2. \tag{27}$$

Now the exact distribution of $F^{(1)}$ is given in [12], since this corresponds to the duration of an unsuccessful transmission period in a fully-connected environment with m users. That is, if Y is the transmission start time of the last colliding packet in an unsuccessful transmission period, we have

$$F^{(1)} = 1 + a + Y$$

$$\text{Prob }[Y \leqslant y] = \frac{(1-e^{-gy}+e^{-ga})^{m-1}-\gamma_2}{1-\gamma_2} \qquad 0 \leqslant y \leqslant a \tag{28}$$

and so

$$E[\{F^{(1)}\}^k] = \overline{(1+a+Y)^k}$$

$$E[Y^k] = \frac{1}{1-\gamma_2}\int_0^a (ky^{k-1})[1-(1-e^{-gy}+e^{-ga})^{M-1}]\,dy \qquad k=1,\ 2. \tag{29}$$

It remains for us to find the distribution of $F^{(2)}$ by approximation. Note that $F^{(2)}$ consists of a random number of consecutive transmissions such that the duration of each interval between two successive starts of transmission is less than $1 + a$. Therefore, we have the same intractability as in the analysis of pure ALOHA which has forced us to the approximation in (16)–(18). In addition, since each CSMA user does not behave independently once any transmission has started (he stops transmission initiations when he hears other transmissions), the independence of source processes in a superposition on which our approximation in pure ALOHA was based is no longer applicable. Nevertheless, we here introduce another assumption for approximation, saying that the intervals between two successive transmission start times at each user are independent and identically distributed as given by (8) but with properly reduced transmission rates g'. We propose that the reduced rates g' be determined as follows:

$g' = g \cdot$ Prob [a user does not hear the ongoing transmission(s)|at least one other user is transmitting]

$$= g \cdot \frac{\left(\dfrac{g^{-1}}{1+a+g^{-1}}\right)^{m-1} - \left(\dfrac{g^{-1}}{1+a+g^{-1}}\right)^{M-1}}{1-\left(\dfrac{g^{-1}}{1+a+g^{-1}}\right)^{M-1}} \tag{30}$$

where the factor $g^{-1}/(1 + a + g^{-1})$ is the probability that a user is not transmitting under the assumption that he (independently of others) alternates between the transmission of length $1 + a$ and the exponentially distributed idle time (with mean $1/g$). Note that if a user is completely hidden from all other users ($m = 1$), then (30) duly gives $g' = g$. On the other hand, if he can be heard by all others ($m = M$), then (30) yields $g' = 0$ which is fine again. Between these extreme cases, (30) gives g' between 0 and g depending on the connectivity m of a user. The more he is heard, the closer g' is to 0.

From the above-mentioned twofold approximations, we may write down the expressions to calculate the distribution of $F^{(2)}$ similar to the treatment in Section IV but with a modification due to the nonzero value of a.

$$F^{(2)} = \sum_{n=1}^{L} f^{(n)} + 1 + a \tag{31}$$

where

$$\text{Prob }[L=n] \cong (1-\delta)^{n-1}\delta \qquad n=1,\ 2,\ \cdots$$
$$\delta = [1+(1+a)g']^{-(M-1)} \tag{32}$$

and

$$\text{Prob }[f^{(n)} > x] = \frac{[1+g'(1+a-x)]^{M-1}-1}{[1+g'(1+a)]^{M-1}-1} \qquad 0 \leqslant x \leqslant 1+a. \tag{33}$$

From (31)–(33), we have the following expressions for the mean and variance of $F^{(2)}$.

$$E[F^{(2)}] = \frac{g_1^M - 1 - (1+a)g'Mg_1^{-(M-1)}}{g'M[1-g_1^{-(M-1)}]}$$

$$\begin{aligned}\text{Var }[F^{(2)}] = {} & \frac{2g_1^{M-1}(g_1^{M+1}-1)}{(g')^2M(M+1)(g_1^{M-1}-1)} + \frac{g_1^{M-1}(g_1^M-1)^2(g_1^{M-1}-2)}{(g'M)^2(g_1^{M-1}-1)^2} \\ & + \frac{(1+a)^2g_1^{M-1}}{(g_1^{M-1}-1)^2}\left[\frac{2g_1^{M-1}}{M}-1\right] - \frac{2(1+a)g_1^{M-1}(g_1^M-1)}{g'M(g_1^{M-1}-1)}\end{aligned} \tag{34}$$

where

$$g_1 \triangleq (1+a)g' + 1. \tag{35}$$

Thus, we can calculate $\bar{F}$ and Var $[F]$ by (27). All other quantities needed to evaluate (5) have also been given.

In the special case $a = 0$, we have the distribution of I given by (13), and

$$\gamma = e^{-(M-m)g}$$

$$g' = g \cdot \frac{(1+g)^{-(m-1)} - (1+g)^{-(M-1)}}{1-(1+g)^{-(M-1)}}. \tag{36}$$

Thus, $\bar{F}$ and Var $[F]$ are simply evaluated via (20) and (21), respectively, with g replaced by g'. In the case of fully-connected users ($m = M$), we have $g' = 0$ so that $\bar{F} =$ Var $[F] = 0$ and $\gamma = 1$ (every transmission is successful). Then we have explicitly ($G = gM$)

$$S = \frac{G}{1+G}; \quad C^2 = \frac{1}{(1+G)^2} \tag{37}$$

which is an exact result for the nonpersistent CSMA with no hidden users in the limit of zero propagation delay [7]. Note that the formulation in [14] (which applies the technique of "reduced rate" to both successful and unsuccessful transmissions without distinguishing the two cases) fails to reach (37) when $M \to \infty$ with $m = M$.

VI. Simulation Results and Discussion

In this section, after describing the simulation model, we compare the simulation results with those calculated by using our approximation. Then some of our basic assumptions for the approximation are discussed with regard to their rationale. At the same time, suggestions on the refinement of the present model are made.

In the simulation program, the nonpersistent CSMA protocol is implemented as follows: when a user finds any ongoing

TABLE I
COMPARISON OF SIMULATION AND APPROXIMATION RESULTS FOR THE THROUGHPUT. $S1$ = LOWER BOUND, $S2$ = UPPER BOUND OF 95 PERCENT CONFIDENCE INTERVALS. S = APPROXIMATION. (a) $M = 20$, $m = 1$, $a = 0.5$. (b) $M = 20$, $m = 10$, $a = 0$. (c) $M = 20$, $m = 19$, $a = 0.5$.

G	S1	S2	S
.1000	.07443	.07583	.07468
.1334	.8958	.9136	.9036
.1778	.1048	.1066	.1059
.2371	.1187	.1209	.1188
.3162	.1257	.1274	.1260
.4217	.1228	.1249	.1239
.5623	.1098	.1114	.1102
.7499	.08563	.08780	.08584

(a)

G	S1	S2	S
.1000	.08575	.08732	.08628
.1334	.1083	.1106	.1096
.1778	.1366	.1387	.1372
.2371	.1659	.1685	.1683
.3162	.2003	.2028	.2011
.4217	.2302	.2350	.2325
.5623	.2547	.2595	.2578
.7499	.2697	.2737	.2710
1.000	.2664	.2713	.2669
1.334	.2435	.2478	.2432
1.778	.1990	.2026	.2025
2.371	.1460	.1494	.1525
3.162	.9327	.9543	.1030
4.217	.04911	.05031	.06156

(b)

G	S1	S2	S
.1000	.08161	.08285	.08239
.1334	.1031	.1050	.1034
.1778	.1271	.1294	.1273
.2371	.1520	.1548	.1534
.3162	.1784	.1814	.1797
.4217	.2026	.2056	.2035
.5623	.2203	.2244	.2212
.7499	.2271	.2307	.2289
1.000	.2238	.2274	.2236
1.334	.2023	.2079	.2039
1.778	.1710	.1740	.1714
2.371	.1307	.1330	.1306
3.162	.08931	.09117	.08812
4.217	.05176	.05308	.05110

(c)

transmission at the time of his scheduled transmission, he defers (reschedules) his transmission by an exponentially distributed time after the end of the current transmission (i.e., $1 + a$ after the start of transmission). Given a set of parameter values (a, M, m, and $G = gM$), we collected 2000 interdeparture times and computed from them the sample mean and sample variance for the throughput values S. In Table I, we show the 95 percent confidence intervals for S using 20 such samples, along with the values calculated by our approximate method for three sets of parameters a, M, and m. In all cases, our approximation yields throughput values within or very close to the 95 percent confidence intervals.

In order to examine more cases, we show in Figs. 2-4 simulation results for S and C^2 using only single samples each consisting of 2000 successful transmissions. Fig. 2(a) and (b) for the symmetric hidden-user configurations in unslotted CSMA systems of $M = 20$ users, with zero propagation delay, manifests excellent agreement between our approximation and simulation results over almost the whole range of the offered channel traffic value G in each case of hiddenness. Although agreement for small G is not surprising because we do not have many collisions there anyway, the agreement for large G is noteworthy. (The treatment in [14] claims its applicability only for relatively small G.) Fig. 3(a) and (b) is for the symmetric hidden-user configurations in CSMA with nonzero propagation delays a. The agreement (over the whole range of G) is again excellent for any reasonable value of a. Fig. 4(a) and (b) demonstrates agreement in various combinations of m and a with a fixed value of $G = 1.778$. (This particular value of G was chosen as it differentiates displayed cases well. We get similar agreement for other values of G.) From these comparisons, we may claim the universality of our approximation method in a wide range of system parameters m, a, and G.

Now, let us examine some of our assumptions which have been introduced to make the analysis tractable. In (16) and (31), we have assumed that each $f^{(n)}$, the interval between two successive transmission start times such that it is shorter than the packet transmission time, is independent and identically distributed, while in reality they are not. We have, however, a theoretical rationale for this assumption for a large user population: i.e., the fact that a large number of merged point processes tends to be a Poisson stream with aggregate rate (Palm-Khintchine theorem; see, e.g., [3]). The apparent good agreement of our results with simulation for as many as 20 users endorses this assertion. In our second assumption, to account for the carrier-sensing effects, we have reduced the transmission rates and used them as parameters in the exponential distributions for transmission rescheduling. In reality, however, since we reschedule transmission at indefinite times until we sense the channel idle, the interval between the actual transmissions is likely to be distributed as a random number of exponentially distributed times rather than as a single exponential distribution with reduced rate.

The dependence of $f^{(n)}$'s on each other may be taken into consideration partially as follows. For example, in the case of unslotted CSMA (including pure ALOHA) with zero propagation delay, we know exactly the distribution of $f^{(1)}$:

$$\text{Prob}\,[f^{(1)} \leqslant x] = \frac{1 - e^{-(M-m)gx}}{1 - e^{-(M-m)g}} \qquad 0 \leqslant x \leqslant 1. \quad (38)$$

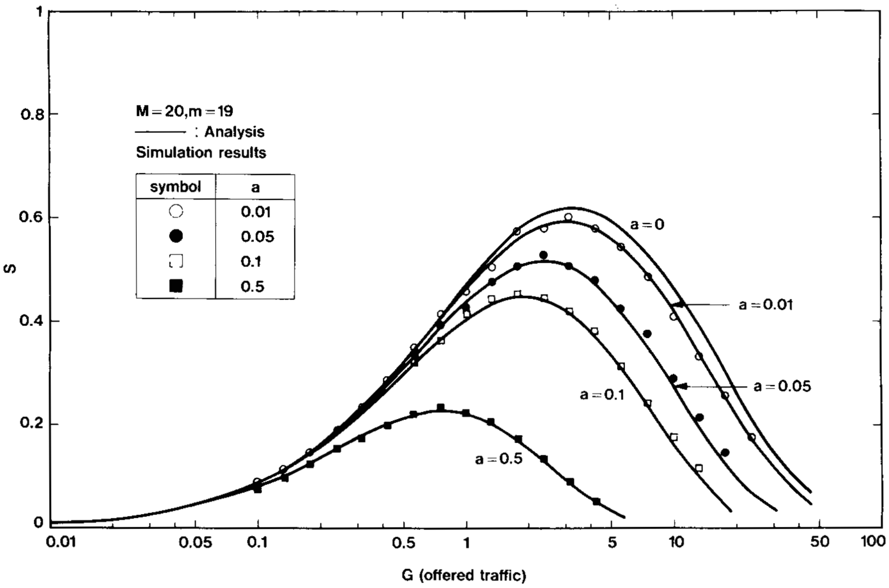
We may also use the (exact) joint distribution of a few successive interevent times given in [5] and [8]. Thus, for those values of offered traffic such that an unsuccessful transmission involves only a few transmissions, this refinement is expected to improve the present formulation results. It is also noted in this connection that Ito [4] has derived the interevent time distribution conditioned on the initial and terminal event sources in the superposed renewal processes. Thus, some improvement based on these ideas can be expected.

VII. CONCLUSION

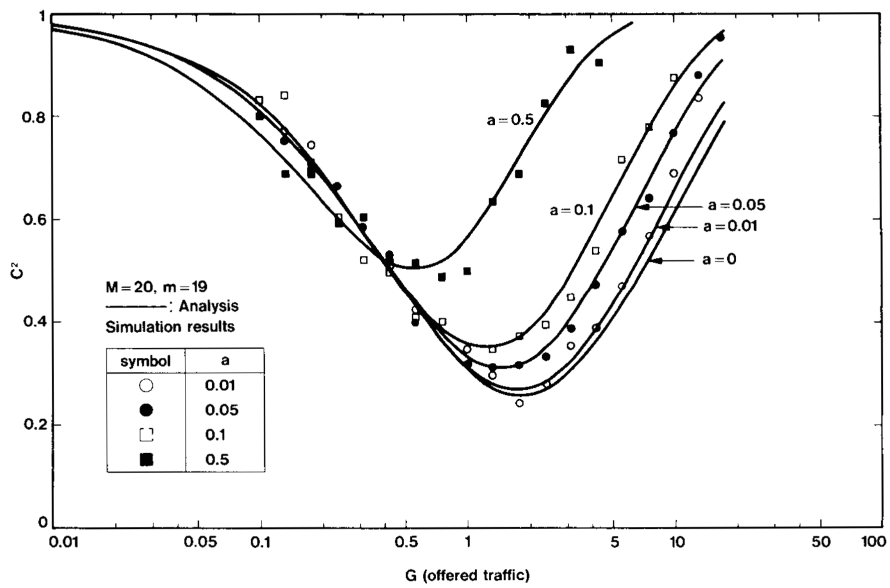
We have given an approximate analysis for the packet departure processes in a hidden-user environment of single-hop packet broadcasting systems. The channel access protocols considered include pure ALOHA and unslotted carrier-sense-multiple-access (CSMA).

Exact stochastic analysis has been given for the durations of a channel idle period, a successful transmission period, and an unsuccessful transmission period consisting only of those packets from the users who can hear the initiating transmission. An approximate analysis has been developed for the duration of an unsuccessful transmission period involving hidden users' packets. Our approximation is based on the theory of superposition of independent renewal processes, together with a proper reduction of transmission start rates to take care of carrier-sense effects.

The channel throughput and the coefficient of variation of the packet interdeparture time calculated by use of our approximation have been compared with the simulation results in symmetric hearing configurations for a variety of degrees of



(a)



(b)

Fig. 3. (a) Throughput (S) and (b) coefficient of variation of the packet interdeparture time (C^2) in unslotted CSMA (nonzero propagation delay) for a symmetric hidden-user configuration ($M = 20$, $m = 19$).

hiddenness. The agreement between them is excellent for almost the whole range of offered channel traffic and all reasonable values of propagation delay. Lastly, we have discussed some rationale for our assumptions and suggested possible refinement.

The first two moments of packet interdeparture time are used in [11] to determine the coefficients in the diffusion process approximation to the queue length distribution at the users.

Appendix

Derivation of (22)

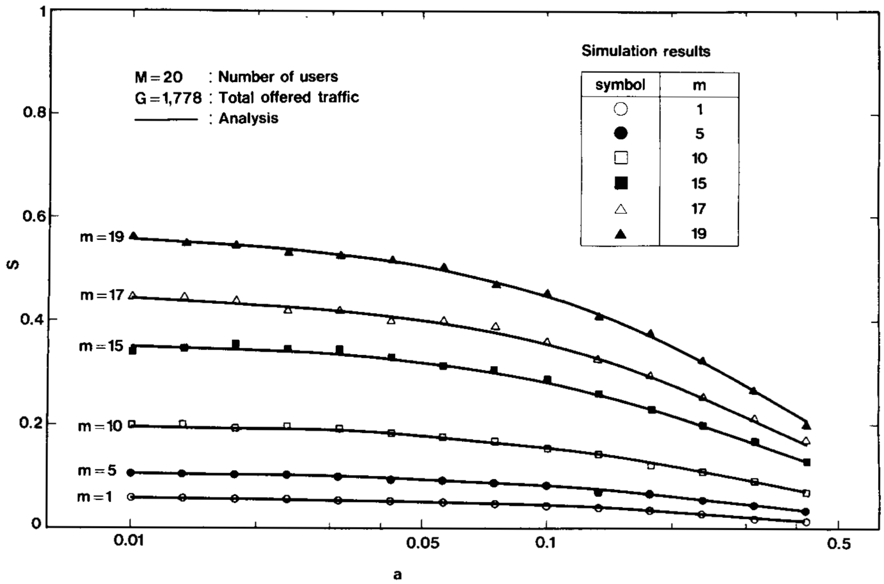
We will derive the expression for $\bar{F}$ in (22), the exact mean duration of an unsuccessful transmission period in pure ALOHA systems. Since each user alternates between the transmitting state of duration 1 and the idle state of mean duration $1/g$, the probability P_0 that the channel is idle at random time is given by

$$P_0 = \left(\frac{1/g}{1+1/g}\right)^M = \left(\frac{1}{1+g}\right)^M. \quad \text{(A.1)}$$

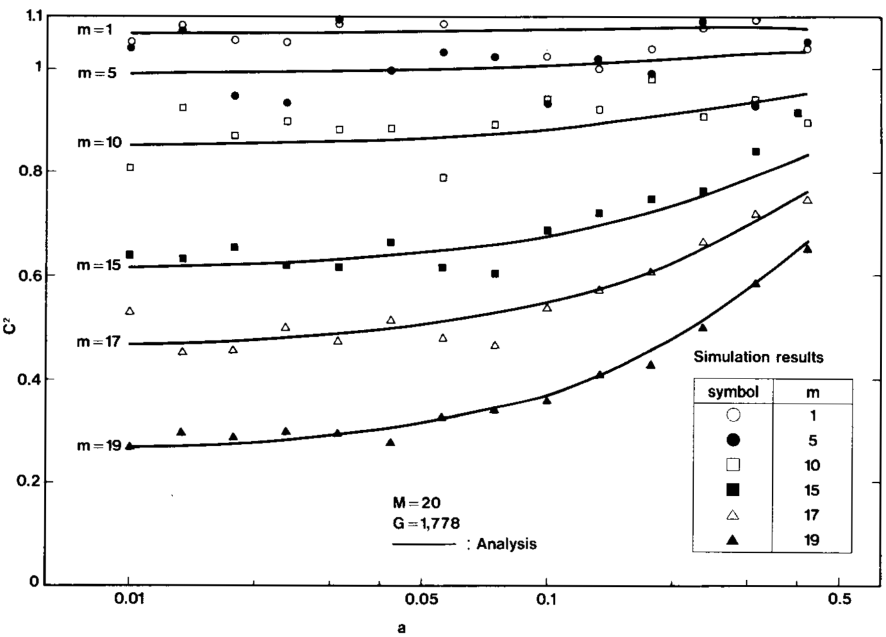
Now, due to the renewal property of channel state, P_0 must equal the ratio of the average duration of channel idle period $\bar{I}$ to the average cycle time $\bar{B} + \bar{I}$, where $\bar{B}$ is the average duration of the common channel busy period:

$$P_0 = \frac{\bar{I}}{\bar{B}+\bar{I}} \quad \text{(A.2)}$$

where $\bar{I}$ is given in (13). Since the duration of a channel busy period is 1 with probability $\gamma = e^{-g(M-1)}$ (a successful transmission period), and is F with probability $1 - \gamma$ (an



(a)



(b)

Fig. 4. (a) Throughput (S) and (b) coefficient of variation of the packet interdeparture time (C^2) in unslotted CSMA for symmetric hidden-user configurations with various a and m.

unsuccessful transmission period), we have a relationship:

$$\bar{B}=e^{-g(M-1)}\cdot 1+[1-e^{-g(M-1)}]\cdot \bar{F}. \qquad \text{(A.3)}$$

Equations (A.1)–(A.3) and (13) yield (22).

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Hideaki Takagi (S'80-M'83) received the B.S. and M.S. degrees in physics from the University of Tokyo, Tokyo, Japan, in 1972 and 1974, respectively, and the Ph.D. degree in computer science from the University of California, Los Angeles, in 1983.

In 1974 he joined IBM Japan, Ltd., as a Systems Engineer. From 1979 to 1983, he was with U.C.L.A., supported by the IBM Japan Overseas Scholarship Program and by a contract from the Defense Advanced Research Projects Agency. Since July 1983, he has been with the IBM Japan Science Institute, Tokyo, where he is currently Manager of Distributed Systems. His research interests include probability theory, queueing theory, and stochastic processes as applied to computer communication networks.

Dr. Takagi is a member of the Association for Computing Machinery, the Operations Research Society of America, the Institute of Electronics and Communication Engineers of Japan, and the Information Processing Society of Japan. He is Managing Editor of *Performance Evaluation.*

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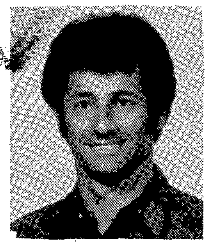

Leonard Kleinrock (S'55-M'64-SM'71-F'73) received the B.S. degree in electrical engineering (evening session) from the City College of New York, New York, NY, in 1957, and the M.S. and Ph.D. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1959 and 1963, respectively.

While at M.I.T., he worked at the Research Laboratory of Electronics as well as with the Computer Research Group of Lincoln Laboratory, Lexington, MA, in advanced technology. He joined the faculty at the University of California, Los Angeles, in 1963, where he is a Professor of Computer Science and Co-Director of the Center for Experimental Computer Science. He is the Principal Investigator for the ARPA Advanced Teleprocessing Systems contract at U.C.L.A. In addition, he is Founder and President of Technology Transfer Institute, Santa Monica, CA, a computer/communications seminar and consulting organization, and is a well-known lecturer in the computer industry. His research interests focus on local area networks, computer networks, performance evaluation, and distributed systems. He has had over 145 papers published and is the author of three books, *Communication Nets: Stochastic Message Flow and Delay* (New York: McGraw-Hill, 1964), *Queueing Systems, Vol. I: Theory* (New York: Wiley, 1975), and *Queueing Systems, Vol. II: Computer Applications* (Wiley, 1976), as well as the *Solutions Manual for Queueing Systems, Vol. I* (Technology Transfer Institute, 1982) and *Vol. II* (to be published, 1986).

Dr. Kleinrock is a Guggenheim Fellow and a member of the National Academy of Engineering and the IBM Science Advisory Committee. He has received numerous best paper and teaching awards, including the ICC 1978 Prize Winning Paper Award, the 1976 Lanchester Prize for outstanding work in operations research, and the Communications Society 1975 Leonard G. Abraham Prize Paper Award. In 1982, as well as having been selected to receive the C.C.N.Y. Townsend Harris Medal, he was co-winner of the L. M. Ericsson Prize, presented by His Majesty, King Carl Gustaf of Sweden, for his outstanding contribution in packet switching technology.