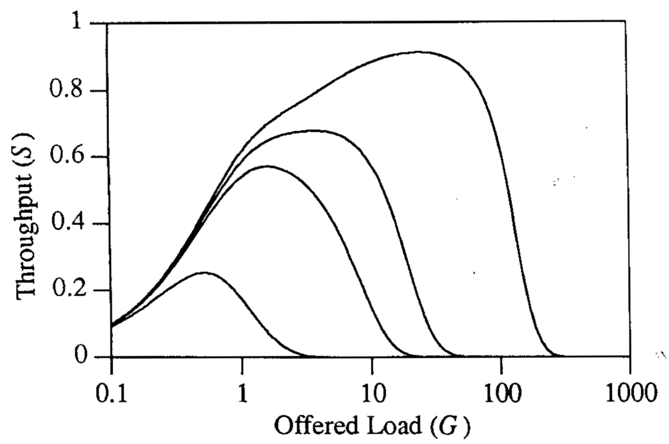

Fig. 3. Throughput versus load in asynchronous CSMA/CD ($a = 0.01$, $b + a = 1.0, 0.1, 0.05, 0.01$).

The stationary probabilities are then easily found to be

$$\pi_1 = (P_{20} + P_{21})/K \tag{17}$$

$$\pi_2 = (1 - P_{10} - P_{11})/K \tag{18}$$

$$\pi_0 = 1 - \pi_1 - \pi_2 = ((1 - P_{11})P_{20} + P_{10}P_{21})/K, \tag{19}$$

where

$$K = (1 - P_{10} - P_{11})(1 + P_{20}) + (1 + P_{10})(P_{20} + P_{21}). \tag{20}$$

Substituting (11)–(20) into (1) provides us with the throughput equation for 1-Persistent CSMA/CD, namely

$$S = \frac{(P_{20} + P_{21})e^{-aG}}{\dfrac{(1 - P_{11})P_{20} + P_{10}P_{21}}{G} + \left((1 - e^{-aG})\left(2a + b + \dfrac{1}{G}\right) + e^{-aG}\right)[P_{20} + P_{21}] + (2a + b)[1 - P_{10} - P_{11}]}$$

V. Results and Discussion

Fig. 3 shows the correct results for throughput, S, as a function of load, G, for the same parameter values as [1, fig. 8], namely $a = 0.01$, and $b + a \in \{1, 0.1, 0.05, 0.01\}$. The resulting behavior is similar to the slotted version of the protocol as shown in [3]. With collision detection, the protocol is able to maintain throughput near capacity over a large range of loads.

We note that in [1], the approach from [3] is extended to include collision detection. However, they do not distinguish between unsuccessful subbusy periods that begin with one or more than one transmission, respectively. In all cases they assume that their duration is given by $b + a + Y_1$, where Y_1 is the transmission time of the first colliding packet as in our case. This oversight seems to be responsible for a number of anomalous results with their model, such as bimodal throughput versus load curves, and the prediction (contradicted by experimental data [5]) that as a and b approach zero, collision detection does not have any effect on the performance and a maximum throughput (i.e., capacity) of only 53 percent is achievable (like 1-Persistent CSMA [2]).

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Correction to "Throughput Analysis for Persistent CSMA Systems"

HIDEAKI TAKAGI AND LEONARD KLEINROCK

This paper corrects errors in Sections IV and VI of [1]. Accordingly, Figs. 6–8 of [1] are also corrected. All terminology and notation below are carried over from [1]. An error in [1] for the special case of unslotted 1-persistent CSMA with collision detection is pointed out by [2] which also corrects the error for the infinite population case using a different approach.

First, we reconsider Section IV of [1], i.e., unslotted p-persistent CSMA. In (36) of [1], we have the probability of the event $\{R > x,\ N(x) = n + m \mid N(0) = n\}$. Since a transmission starts with probability $(n + m)pdx$ during dx after this event, we get

$$\begin{aligned}\text{Prob}\,[x < R \le x + dx,\ N(x) = n + m \mid N(0) = n] &= e^{-pnx}\binom{M-n}{m} e^{-gx(M-n-m)} \\ &\times \left[\frac{g(e^{-gx} - e^{-px})}{p - g}\right]^m (n + m)pdx \end{aligned} \tag{1}$$

which leads to (37) and (38) of [1] (the numerator in (37) of [1] should read $pe^{-gx} - ge^{-px}$). Now, we must use (1) to

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H. Takagi is with the IBM Japan Science Institute, Chiyoda-ku, Tokyo 102, Japan.
L. Kleinrock is with the Department of Computer Science, University of California, Los Angeles, CA 90024.

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uncondition (40) and (41) of [1]. Thus, (42), (43), (44) and (46) of [1] are corrected respectively as follows.

$$f(x, y; n) \triangleq \text{Prob}\,[x<R\le x+dx,\ Y\le y\,|\,N(0)=n]/dx$$
$$= npe^{-pnx}(1-e^{-py}+e^{-pa})^{n-1}\left[\frac{pe^{-gx}(1-e^{-gy}+e^{-ga})-ge^{-px}(1-e^{-py}+e^{-pa})}{p-g}\right]^{M-n}$$
$$+(M-n)pe^{-pnx}(1-e^{-py}+e^{-pa})^{n}$$
$$\times\left[\frac{pe^{-gx}(1-e^{-gy}+e^{-ga})-ge^{-px}(1-e^{-py}+e^{-pa})}{p-g}\right]^{M-n-1}\frac{g(e^{-gx}-e^{-px})}{p-g} \qquad (2)$$

$$\gamma_{(n)} \triangleq \int_0^\infty f(x, 0; n)\,dx \qquad (3)$$
$$E[Y_{(n)}]=E[Y\,|\,N(0)=n]$$
$$=a-\int_0^\infty dx\int_0^a f(x, y; n)\,dy \qquad (4)$$
$$p_{nk}=g_k(1)\gamma_{(n)}+\int_0^\infty dx\int_0^a g_k(1+y)\,d_yf(x, y; n) \qquad (5)$$

where $d_yf(x, y; n) \triangleq [\partial f(x, y; n)/\partial y]dy$. Note that $f(x, y; n)$ represents the joint probability distribution for R and Y (Y depends on R). Equations for the unslotted 1-persistent CSMA in Section V of [1] are correct.

In Section VI of [1], unslotted persistent CSMA with collision detection, we must uncondition (56) of [1] by using (1) for the same reason as above. Then, (62) and (63) of [1] should be replaced by

$$B_n=E[R_{(n)}]+\gamma_{(n)}+[1-\gamma_{(n)}](b+a)$$
$$+\int_0^\infty dx\int_0^a f_1(x, y; n)\,dy+\sum_{k=1}^{M}p_{nk}B_k \qquad (6)$$
$$p_{nk} \triangleq \gamma_{(n)}g_k(1)-\int_0^\infty dx\int_0^a g_k(b+y)\,d_yf_1(x, y; n) \qquad (7)$$

where

$$f_1(x, y; n) \triangleq \text{Prob}\,[x<R\le x+dx,\ Y_1>y\,|\,N(0)=n]/dx$$
$$=npe^{-pnx-p(n-1)y}\left[\frac{pe^{-g(x+y)}-ge^{-p(x+y)}}{p-g}\right]^{M-n}$$
$$+(M-n)pe^{-pn(x+y)}$$
$$\times\left[\frac{pe^{-g(x+y)}-ge^{-p(x+y)}}{p-q}\right]^{M-n-1}$$
$$\times\frac{g(e^{-gx}-e^{-px})}{p-q} \qquad (8)$$

Note that $\gamma_{(n)} = \int_0^\infty f_1(x, a; n)dx$ is identical to (3).

In unslotted 1-persistent CSMA with collision detection, the transition probabilities $\{p_{nk}\}$ are given by

$$n=1:\ p_{1k}=e^{-g(M-1)a}g_k(1)$$
$$+g(M-1)\int_0^a g_k(b+y)e^{-g(M-1)y}\,dy$$
$$n>1:\ p_{nk}=\binom{M}{k}(1-e^{-bg})e^{-bg(M-k)} \qquad k=0, 1, \cdots, M \qquad (9)$$

Using them, the system of equations for $\{B_n\}$ and $\{U_n\}$ is given by

$$n=1:\ B_1=e^{-g(M-1)a}$$
$$+[1-e^{-g(M-1)a}]\left[b+a+\frac{1}{g(M-1)}\right]+\sum_{k=1}^{M}p_{1k}B_k$$
$$n>1:\ B_n=b+a+\sum_{k=1}^{M}p_{nk}B_k \qquad (10)$$
$$n=1:\ U_1=e^{-g(M-1)a}+\sum_{k=1}^{M}p_{1k}U_k$$
$$n>1:\ U_n=\sum_{k=1}^{M}p_{nk}U_k \qquad (11)$$

By solving them, we get B_1 and U_1 which we use in (34) of [1].

In the limit of $M\to\infty$ with $G = gM$ fixed at a finite value, p_{10} and p_{11} become

$$p_{10}=e^{-G(1+a)}+\frac{1}{2}e^{-bG}(1-e^{-2aG})$$
$$p_{11}=Ge^{-G(1+a)}$$
$$+Ge^{-bG}\left[\left(\frac{b}{2}+\frac{1}{4G}\right)(1-e^{-2aG})-\frac{a}{2}e^{-2aG}\right] \qquad (12)$$

(These are identical to (68) and (69) in [1]. Also, p_{nk} for $n > 1$ becomes independent of n:

$$n>1:\ p_{nk}=\frac{(bG)^k}{k!}e^{-bG} \qquad k=0, 1, 2, \cdots \qquad (13)$$

Thus, we get

$$n=1:\ B_1=\frac{(p_{20}+p_{21})[e^{-aG}+(1-e^{-aG})(b+a+1/G)]+(1-p_{10}-p_{11})(b+a)}{p_{20}(1-p_{11})+p_{21}p_{10}}$$
$$n>1:\ B_n=\frac{b+a+p_{21}B_1}{p_{20}+p_{21}} \qquad (14)$$
$$n=1:\ U_1=\frac{(p_{20}+p_{21})e^{-aG}}{p_{20}(1-p_{11})+p_{21}p_{10}}$$
$$n>1:\ U_n=\frac{p_{21}U_1}{p_{20}+p_{21}} \qquad (15)$$

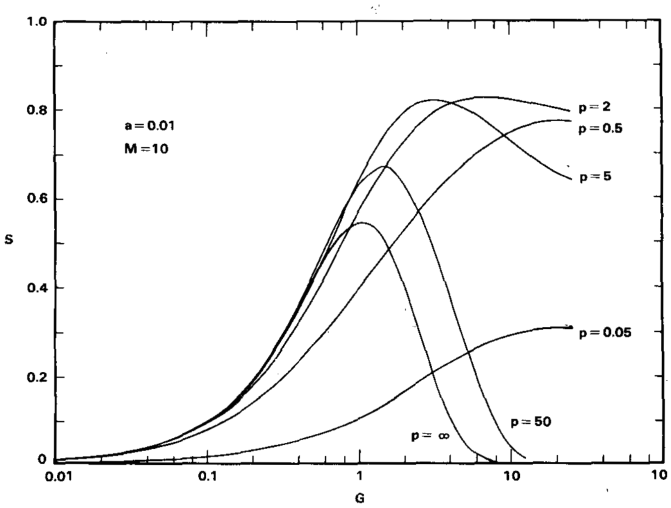

Fig. 1. Throughput of unslotted p-persistent CSMA.

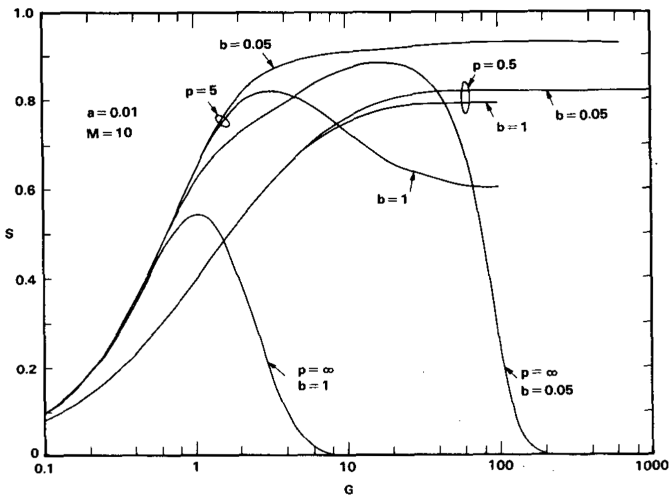

Fig. 2. Throughput of unslotted p-persistent CSMA with collision detection.

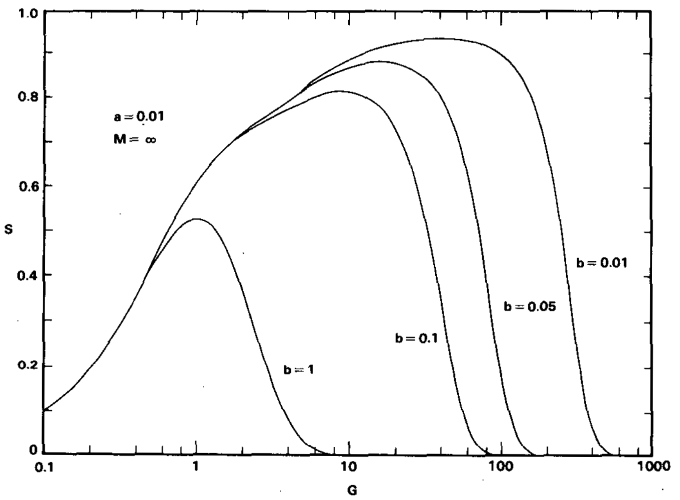

Fig. 3. Throughput of unslotted 1-persistent CSMA with collision detection.

Using these expressions in (34) of [1], we obtain

$$S=\frac{(p_{20}+p_{21})e^{-aG}}{\frac{1}{G}[(2-p_{11})p_{20}+(1+p_{10})p_{21}]-e^{-aG}\left(a+b+\frac{1}{G}-1\right)(p_{20}+p_{21})+(b+a)(1-p_{10}-p_{11}+p_{20}+p_{21})} \tag{16}$$

(this is identical to the result in [2]) or, explicitly,

$$S=\frac{G(1+bG)e^{-(a+b)G}}{\left\{\begin{array}{l} G(a+b)[1-(1+G)e^{-(1+a)G}]-(1+bG)[1-G(1-a-b)]e^{-(a+b)G} \\ \quad +e^{-bG}\left[2+\frac{G}{4}(a+5b)+\frac{1}{2}b(a+b)G^2\right]+\frac{G}{4}(2bG+3+2aG)(a+b)e^{-(2a+b)G} \\ \quad -G(1-b)e^{-(1+a+b)G}-\frac{1}{4}e^{-2bG}[1-(1+2aG)e^{-2aG}] \end{array}\right\}} \tag{17}$$

According to these corrections, Figs. 6–8 in [1] are redrawn here as Figs. 1–3, respectively. (For numerical evaluation of (3), (4), etc., the range of integration $[0, \infty]$ in x is transformed to a finite range $[0, 1]$ in z by replacement $z = e^{-x}$.) In the case of p-persistent CSMA ($p < \infty$), some apparent differences for large values of $G = gM$ are due to numerical instability; otherwise there seems little discernible difference due to the above correction. However, for 1-persistent CSMA with collision detection, the curve for $p = \infty$, $b = 0.05$ in Fig. 7 of [1] is appreciably corrected in Fig. 2. Similarly, the appearance of Fig. 8 of [1] and Fig. 3 is greatly different. As pointed out in [2], the double-hump anomaly in Fig. 8 of [1] does not appear in Fig. 3.

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