Throughput Analysis for Persistent CSMA Systems

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Abstract—**The channel throughput for a finite number of packet broadcasting users is analyzed for random access protocols, including slotted persistent carrier sense multiple access (CSMA) with and without collision detection and unslotted persistent CSMA with and without collision detection. We consider both *p*- and 1-persistent CSMA. Our results can be extended to infinite population cases (by taking the proper limit), where they agree with the known throughput expressions when available.**

I. INTRODUCTION

ONE class of multiaccess protocols for packet communication systems is the random access (or contention) technique, where the entire bandwidth is provided to the users as a single channel to be accessed randomly. Carrier sense multiple access [6] is included in this category. Here one of the basic measures for the efficiency of protocol is the throughput, i.e., the average fraction of time that the channel is used for useful data communication. (Three factors accounting for the throughput degradation are propagation delay, user's idle [not transmitting] period, and packet collision [overlapping of transmissions from multiple users] inherent in the random access.)

This paper focuses on the throughput analysis for CSMA-type random access protocols for a finite number of users (transmitters) and a single receiver in line-of-sight of all users. A number of studies on the throughput analysis for random access protocols have already appeared in the literature. However, most of them have been based on the assumption that there are infinitely many users, such that the collective channel traffic forms a Poisson process with a finite rate. Thus, our study gives useful information for ground packet radio systems as well as local area computer networks, which consist of a relatively small number of users. (A system with a relatively large number of users can be approximated by an infinite population model.) The analysis for finite-population systems may also serve as the first step to approach the performance modeling of multihop packet radio networks (such as PRNET described in [4]), where each user has only a limited number of communicating neighbors.

The channel access protocols we consider here are

1) slotted p- and 1-persistent CSMA
2) slotted p- and 1-persistent CSMA with collision detection.
3) unslotted p- and 1-persistent CSMA
4) unslotted p- and 1-persistent CSMA with collision detection.

(Each protocol model is described individually below.) Previous work based on the infinite population model for the subset of the above-mentioned protocols includes [6] (slotted p- and 1-persistent CSMA and unslotted 1-persistent CSMA) and [7] (slotted 1-persistent CSMA with collision detection). In this paper, we give an exact throughput analysis in the case of a finite population for all of the above-listed protocols. Arthurs and Stuck [1] also provide an analysis of throughput for slotted and unslotted persistent CSMA with collision detection based on models different from ours. (A difference between our model and theirs involves the way in which one disposes of a packet, the transmission of which is suppressed as a result of packets sensing a busy channel. In our model, similar to [6], they are dismissed from the system at the beginning of the next transmission period, while in [1] they are retained in the buffer until successfully transmitted; so 1-persistent CSMA is impossible in the [1] model.)

In dealing with the case of a finite population, we *assume* that each user has periods, which are independent and exponentially or geometrically (depending on whether the time is continuous or slotted) distributed, in which he has no packets. By superimposing these idle periods over all users, the system idle period in which no users have a packet (denoted by I) is easily seen also to be exponentially or geometrically distributed. This assumption makes analysis tractable by taking advantage of the memoryless property [5, p. 66]. The case of an infinite population does not need a specific assumption on the distribution of each user's idle period because the Palm-Khinchine theorem (see [3]) guarantees that the collective idle period is always independent and exponentially or geometrically distributed.

Due to the above assumption, we recognize that each epoch in the system idle period is a regenerative point, in the sense that the system state after any such epoch is a probabilistic replica of the system state beginning at the previous such epoch. Thus, the system state alternates between idle periods I and busy periods B in which at least one user has a packet. We call consecutive pair B and I a *regeneration cycle.* Let U be the time spent in useful transmission during a regeneration cycle. Then, the channel throughput S is generally expressed as

$$S = \frac{\bar{U}}{\bar{B} + \bar{I}} \tag{1}$$

where $\bar{X}$ denotes the expectation of a random variable X, i.e., $\bar{X} \triangleq E[X]$.

Throughout the paper we assume a constant packet length whose transmission time is chosen as the unit of time. We denote by M the number of users and by G the total packet arrival rate (in units of packets per packet transmission time). We take into account the signal propagation delay denoted by a.

Each section below begins with a description of the protocol model, followed by the definition of protocol-dependent parameters. The condition for a successful transmission is stated. Then, we evaluate $\bar{I}$, $\bar{B}$, and $\bar{U}$ for the finite population model. Our result in the limit $M \to \infty$ is shown to be in agreement with the known results for an infinite population.

II. SLOTTED PERSISTENT CSMA

In slotted CSMA we assume the time is slotted with slot size a (the propagation delay), and all users are synchronized to start transmission only at slot boundaries. (For convenience, we assume that $1/a$ is an integer.) An attempted transmission is successful if none of the other users start a transmission at

Paper approved by the Editor for Computer Communication of the IEEE Communications Society for publication without oral presentation. Manuscript received May 9, 1983; revised February 13, 1985. This work was supported by the Defense Advanced Research Projects Agency under Contract MDA 903-82-C-0064.

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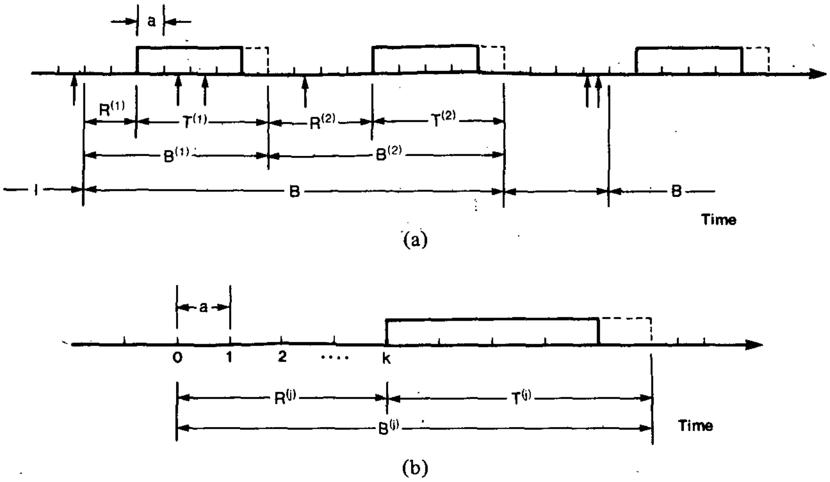

Fig. 1. Channel state in slotted persistent CSMA. (a) Overview of channel state. (b) Close look at the jth sub(busy)period.

the same time. For the following $1/a$ slot boundaries, all other users suppress the start of their transmission due to carrier sensing. Thus, the duration of a transmission period (whether successful or not) is $1 + (1/a)$ slots. We only consider the case of statistically identical users. In persistent CSMA [6], all the packets that arrive during any ongoing transmission are buffered until some next transmission is started in the channel, and then they are discarded or rescheduled. Let each empty user (who does not have a buffered packet) have an arrival with probability g (and does not with probability $1 - g$) in any slot, where $0 < g < 1$. (This we call a geometric arrival process.) Assume that each nonempty user (who has a buffered packet) starts transmission with probability p (and does not with probability $1 - p$) at the slot boundaries following any idle slot. (This is a slotted p-persistent protocol, where $0 < p \leqslant 1$.) Our derivation of channel throughput for a finite population model follows the approach in [6] for an infinite population model.

To analyze the throughput, let us introduce some notation which defines the channel states as illustrated in Fig. 1(a). Let a channel idle period (denoted by I) be the time in which the channel is idle and no packets are awaiting transmission. When any packet arrives, the next slot is said to begin a busy period (denoted by B) which ends if no packets have accumulated at the end of transmission. Let U be the time spent for successful transmission(s) in a busy period B. Then, we have the channel throughput as in (1).

We divide a channel busy period into several sub(busy)periods such that the jth subperiod (denoted by $B^{(j)}$) consists of a transmission delay (denoted by $R^{(j)}$) followed by a transmission time (denoted by $T^{(j)}$); see Fig. 1(b). A transmission delay is the time in which the channel is idle and packets are awaiting transmission; in the 1-persistent protocol, $R^{(j)}$ is always zero since packets start transmission as soon as they arrive. In CSMA without collision detection, we have $T^{(j)} = 1 + a$ whether the transmission is successful or not. Thus, we have

$$B(j) = R(j) + 1 + a \qquad j = 1, 2, \cdots. \tag{2}$$

Finally, let $U^{(j)}$ be the useful transmission time in the jth subperiod:

$$U^{(j)} = \begin{cases} 1 & \text{if } T^{(j)} \text{ is successful} \\ 0 & \text{if } T^{(j)} \text{ is unsuccessful.} \end{cases} \tag{3}$$

Next, let J be the number of sub(busy)periods included in a busy period B. Then we have

$$B = \sum_{j=1}^{J} B^{(j)}; \quad U = \sum_{j=1}^{J} U^{(j)}. \tag{4}$$

Since the busy period continues as long as there is at least one arrival during the last transmission time (such an event occurs with probability $1 - (1 - g)^{(1+(1/a))M}$), J is geometrically distributed as

$$\text{Prob}\,[J = j] = [1 - (1 - g)^{(1+(1/a))M}]^{j-1} \cdot (1 - g)^{(1+(1/a))M} \qquad j = 1, 2, \cdots$$

$$\bar{J} = \frac{1}{(1 - g)^{(1+(1/a))M}}. \tag{5}$$

Note in (4) that $\{B^{(j)}; j = 1, 2, \cdots, J\}$ are independent and $\{B^{(j)}; j = 2, 3, \cdots, J\}$ are identically distributed. Also, J is independent of each $B^{(j)}$. The same thing can be said for $\{U^{(j)}\}$. Thus, we have

$$\bar{B} = E[B^{(1)}] + (\bar{J} - 1)E[B^{(2)}];$$
$$\bar{U} = E[U^{(1)}] + (\bar{J} - 1)E[U^{(2)}]. \tag{6}$$

It is clear that the duration of an idle period I is geometrically distributed as

$$\text{Prob}\,[I = ka] = (1 - g)^{M(k-1)}[1 - (1 - g)^M] \qquad k = 1, 2, \cdots$$

$$\bar{I} = a/[1 - (1 - g)^M]. \tag{7}$$

Thus, from (1), (2), and (5)-(7), we can calculate S if we know $E[R^{(j)}]$ and $E[U^{(j)}]$ for $j = 1, 2$.

Let $\pi_n(X)$ be the probability that we have n arrivals among M users in X slots, given that $n \geqslant 1$. Using the geometric arrival rate g at each of M users in a slot, we have

$$\pi_n(X) = \frac{1}{1 - (1 - g)^{MX}} \binom{M}{n} [1 - (1 - g)^X]^n \cdot (1 - g)^{X(M-n)} \qquad n = 1, 2, \cdots, M. \tag{8}$$

Since $X = 1$ for $j = 1$ and $X = T^{(j-1)}/a = 1 + (1/a)$ for $j \geqslant 2$, the distribution of the number of packets awaiting transmission at the beginning of $B^{(j)}$, denoted by $N_0^{(j)}$, is given by

$$\text{Prob}\,[N_0^{(j)} = n] = \begin{cases} \pi_n(1) & j = 1 \\ \pi_n(1 + (1/a)) & j = 2, 3, \cdots. \end{cases} \quad (9)$$

[Note in (9) that $N_0^{(j)}, j \geq 2$, is assumed to be independent of the number of nonempty users at the beginning of $T^{(j-1)}$, which may not be true, depending on when a user which has a packet at the beginning of $T^{(j-1)}$ discards it and can thus accept the next arrival. However, for simplicity we assume that all users make room for new arrivals from the beginning of $T^{(j-1)}$ by putting aside the already buffered packets (including the packets being transmitted in $T^{(j-1)}$) which are to be discarded; this is the same assumption as in [6] and leads to (9).]

The distribution of $R^{(j)}$, given $N_0^{(j)} = n$, $j \geq 1$, can be found as follows. Let us number the slot boundaries as $k = 0, 1, 2, \cdots$ from the beginning of $R^{(j)}$, as depicted in Fig. 1(b), and denote by $N_k^{(j)}$ the number of awaiting packets at the kth boundary. Then, according to the p-persistent protocol and geometric arrivals, we have ($A_i^{(j)}$ being the number of arrivals during the ith slot in $R^{(j)}$)

$$\begin{aligned} &\text{Prob}\,[R^{(j)} \geq ka,\ A_i^{(j)} = m_i (1 \leq i \leq k) \mid N_0^{(j)} = n] \\ &= (1-p)^n \binom{M-n}{m_1} g^{m_1}(1-g)^{M-n-m_1} \\ &\cdot (1-p)^{n+m_1} \binom{M-n-m_1}{m_2} \\ &\cdot g^{m_2}(1-g)^{M-n-m_1-m_2} \cdots (1-p)^{n+m_1+\cdots+m_{k-1}} \\ &\cdot \binom{M-n-m_1-\cdots-m_{k+1}}{m_k} \\ &\cdot g^{m_k}(1-g)^{M-n-m_1-\cdots-m_k} \\ &= (1-p)^{kn}(1-g)^{k(M-n)}(M-n)! \\ &\cdot \prod_{i=1}^{k} \frac{1}{(m_i)!} \left[\frac{g(1-p)^{k-i}}{(1-g)^{k+1-i}}\right]^{m_i} \end{aligned} \quad (10)$$

It follows that

$$\begin{aligned} &\text{Prob}\,[R^{(j)} \geq ka, N_k^{(j)} = n + m \mid N_0^{(j)} = n] \\ &= \sum_{\sum_{i=1}^{k} m_i = m} \text{Prob}\,[R^{(j)} \geq ka, A_i^{(j)} = m_i (1 \leq i \leq k) \mid N_0^{(j)} = n] \\ &= (1-p)^{kn}(1-g)^{k(M-n)} \binom{M-n}{m} \left[\sum_{i=1}^{k} \frac{g(1-p)^{k-i}}{(1-g)^{k+1-i}}\right]^m \\ &= (1-p)^{kn}(1-g)^{k(M-n)} \binom{M-n}{m} \left(\frac{g}{p-g}\right)^m \\ &\cdot \left[1 - \left(\frac{1-p}{1-g}\right)^k\right]^m \qquad m = 0, 1, 2, \cdots, M-n. \end{aligned} \quad (11)$$

From (11), the expected value of $R^{(j)}$, given $N_0^{(j)} = n, j \geq 1$, is evaluated as

$$\begin{aligned} &E[R^{(j)} \mid N_0^{(j)} = n] \\ &= a \sum_{k=1}^{\infty} \sum_{m=0}^{M-n} \text{Prob}\,[R^{(j)} \geq ka, N_k^{(j)} = n + m \mid N_0^{(j)} = n] \\ &= a \sum_{k=1}^{\infty} (1-p)^{kn} \left[\frac{p(1-g)^k - g(1-p)^k}{p-g}\right]^{M-n}. \end{aligned} \quad (12)$$

Unconditioning (12) by using (8) and (9), we get

$$E[R^{(j)}] = \begin{cases} r(1) & j = 1 \\ r(1 + (1/a)) & j = 2, 3, \cdots \end{cases} \quad (13)$$

where

$$\begin{aligned} r(X) \triangleq &\frac{a}{1-(1-g)^{XM}} \sum_{k=1}^{\infty} \left\{(1-p)^k - (1-g)^X p \right. \\ &\left. \cdot \left[\frac{(1-p)^k - (1-g)^k}{p-g}\right]\right\}^M \\ &- \frac{a(1-g)^{XM}}{1-(1-g)^{XM}} \sum_{k=1}^{\infty} \left[\frac{p(1-g)^k - g(1-p)^k}{p-g}\right]^M. \end{aligned} \quad (14)$$

Thus, from (2), (5)–(7), (13), and (14), we get

$$\begin{aligned} \bar{B} + \bar{I} &= E[R^{(1)}] + 1 + a + [(1-g)^{-(1+(1/a))M} - 1] \\ &\cdot (E[R^{(2)}] + 1 + a) + \bar{I} \\ &= \frac{1+a}{(1-g)^{(1+(1/a))M}} + \frac{a}{(1-g)^{(1+(1/a))M}} \\ &\cdot \sum_{k=1}^{\infty} \left\{(1-p)^k - (1-g)^{1+(1/a)} p \right. \\ &\left. \cdot \left[\frac{(1-p)^k - (1-g)^k}{p-g}\right]\right\}^M. \end{aligned} \quad (15)$$

We proceed to calculate $E[U^{(j)}]$. From the definition of $U^{(j)}$ in (3), we have

$$\begin{aligned} &E[U^{(j)} \mid R^{(j)} \geq ka, N_k^{(j)} = n + m, \\ &N_0^{(j)} = n] = (n+m)p(1-p)^{n+m-1} \end{aligned} \quad (16)$$

(i.e., the probability of one transmission out of $n + m$ nonempty users). Unconditioning (16) on $R^{(j)}$ and $N_k^{(j)}$ by using (11), we have

$$\begin{aligned} &E[U^{(j)} \mid N_0^{(j)} = n] \\ &= np \sum_{k=0}^{\infty} (1-p)^{(k+1)n-1} \left[\frac{p(1-g)^{k+1} - g(1-p)^{k+1}}{p-g}\right]^{M-n} \end{aligned}$$

$$+ (M-n)gp \sum_{k=1}^{\infty} (1-p)^{(k+1)n} \left[\frac{(1-g)^k - (1-p)^k}{p-g}\right] \cdot \left[\frac{p(1-g)^{k+1} - g(1-p)^{k+1}}{p-g}\right]^{M-n-1}. \tag{17}$$

Further unconditioning (17) on $N_0^{(j)}$ by using (8) and (9), we get

$$E[U^{(j)}] = \begin{cases} u(1) & j = 1 \\ u(1+1/a) & j = 2, 3, \cdots \end{cases} \tag{18}$$

where

$$u(X) \triangleq \frac{pM}{1-(1-g)^{XM}} \sum_{k=0}^{\infty} \left\{(1-p)^k - (1-g)^X \cdot \left[\frac{p(1-p)^k - g(1-g)^k}{p-g}\right]\right\} \left\{(1-p)^{k+1} - (1-g)^X p \left[\frac{(1-p)^{k+1} - (1-g)^{k+1}}{p-g}\right]\right\}^{M-1}$$
$$- \frac{pgM(1-g)^{XM}}{1-(1-g)^{XM}} \sum_{k=1}^{\infty} \left[\frac{(1-g)^k - (1-p)^k}{p-g}\right] \cdot \left[\frac{p(1-g)^{k+1} - g(1-p)^{k+1}}{p-g}\right]^{M-1} \tag{19}$$

From (5), (6), (18), and (19), we obtain

$$\bar{U} = \frac{pM}{(1-g)^{(1+(1/a))M}} \sum_{k=0}^{\infty} (1-p)^k - (1-g)^{1+(1/a)} \cdot \left[\frac{p(1-p)^k - g(1-g)^k}{p-g}\right] \cdot \left\{(1-p)^{k+1} - (1-g)^{1+(1/a)} f \left[\frac{(1-p)^{k+1} - (1-g)^{k+1}}{p-g}\right]\right\}^{M-1} \tag{20}$$

We note that, in 1-persistent CSMA ($p = 1$), we have simply

$$r(X) \equiv 0 \text{ and } u(X) = \frac{M(1-g)^{X(M-1)}[1-(1-g)^X]}{1-(1-g)^{XM}}. \tag{21}$$

The limit $M \to \infty$, with $aG = gM$ held at a fixed value, gives the throughput for an infinite population model. In this case, we use the following expressions for $r(X)$ and $u(X)$ (obtained from (14) and (19) by taking the limit):

$$r(X) = \frac{a}{1-e^{-aGX}} \sum_{k=1}^{\infty} \{\exp[aGX(1-p)^k] - 1\} \cdot \exp\left\{aG\left[-X - k + \frac{1-(1-p)^k}{p}\right]\right\} \tag{22}$$

$$u(X) = \frac{aGp}{1-e^{-aGX}} \sum_{k=0}^{\infty} \left\{\left[(1-p)^k X + \frac{1-(1-p)^k}{p}\right] \cdot \exp[aGX(1-p)^{k+1}] - \frac{1-(1-p)^k}{p}\right\} \cdot \exp\left\{aG\left[-X - (k+1) + \frac{1-(1-p)^{k+1}}{p}\right]\right\}. \tag{23}$$

We further note that, in the 1-persistent case ($p = 1$), we have

$$r(X) \equiv 0 \quad \text{and} \quad u(X) = aGXe^{-aGX}/(1-e^{-aGX}). \tag{24}$$

Substituting (15) and (20) into (1), we get the channel throughput of a slotted p-persistent CSMA system (with propagation delay a) consisting of M identical users, each with geometric arrival rate g:

$$S = \frac{pM \sum_{k=0}^{\infty} \left((1-p)^k - (1-g)^{1+(1/a)} \left[\frac{p(1-p)^k - g(1-g)^k}{p-g}\right]\right) \cdot \left((1-p)^{k+1} - (1-g)^{1+(1/a)} p \left[\frac{(1-p)^{k+1} - (1-g)^{k+1}}{p-g}\right]\right)^{M-1}}{1 + a + a \sum_{k=1}^{\infty} \left((1-p)^k - (1-g)^{1+(1/a)} p \left[\frac{(1-p)^k - (1-g)^k}{p-g}\right]\right)^M} \tag{25}$$

By letting $p = 1$ in (25) or using (21), we have the channel throughput of slotted 1-persistent CSMA as

$$S = \frac{M(1-g)^{(M-1)(1+(1/a))}\{[1-(1-g)^{1+(1/a)}][1-(1-g)^M] + g(1-g)^{M+(1/a)}\}}{(1+a)[1-(1-g)^M] + a(1-g)^{(1+(1/a))M}} \tag{26}$$

In the limit $M \to \infty$ with $aG = gM$ held at a finite value, (25) and (26) become

$$S = \frac{G \sum_{k=0}^{\infty} (p(1-p)^k + a[1-(1-p)^{k+1}]) \exp\left(G(1-p)^{k+1} + aG\left[-(k+1) + \frac{1-(1-p)^{k+2}}{p}\right]\right)}{(1+a)e^{(1+a)G} + a \sum_{k=1}^{\infty} \exp\left(G(1-p)^k + aG\left[-k + \frac{1-(1-p)^{k+1}}{p}\right]\right)}. \tag{27}$$

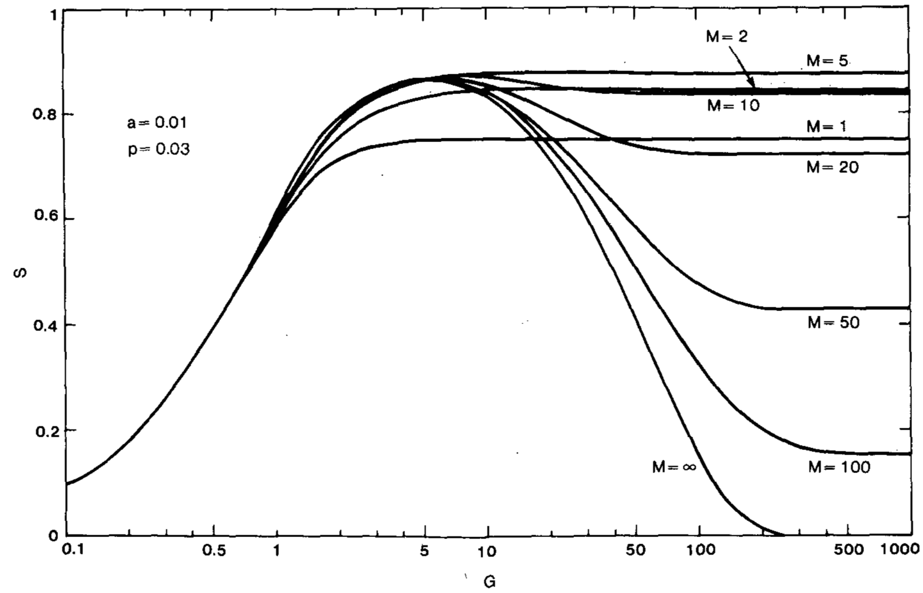

Fig. 2. Throughput of slotted p-persistent CSMA.

and

$$S = \frac{Ge^{-(1+a)G}(1 + a - e^{-aG})}{(1 + a)(1 - e^{-aG}) + ae^{-(1+a)G}} \tag{28}$$

respectively. Equation (28) (an infinite population model of slotted 1-persistent CSMA) is derived in [6], where only a procedure to obtain (27) (an infinite population model of slotted p-persistent CSMA) is also given. An explicit expression of (27) is now provided in this paper. Also provided are (25) and (26) for finite population models.

In Fig. 2, we show the throughput of slotted $p(=0.03)$-persistent CSMA for M users in terms of the total offered traffic rate $G = gM/a$ ($a = 0.01$). We let $g = \min[1, aG/M]$. A few interesting observations here are: i) the maximum throughput values are not very dependent on M as long as $M \geqslant 5$; ii) for a finite M, the throughput does not degrade to zero as G becomes large (the case where the busy period is very long but steadily pushes packets out); iii) the curves we see by increasing M from 1 to ∞ resemble those for the throughput-load relationship in flow-controlled systems (see, for example, [2]). Elaborating on iii) above, we see that the maximal throughput of an uncontrolled system ($M = \infty$) is higher than that of an excessively controlled system ($M = 1$). The throughput of less controlled systems (large M) quickly degrades with congestion (large G), while that of controlled systems does not. The highest maximum throughput and sustained behavior in cases of congestion are achieved in moderately controlled systems (around $M = 5$ here). We note, however, that for $p = 1$ we have only feature i) above; except for $M = 1$ in this case, the throughput equally degrades as G gets large for any number of users.

III. Slotted Persistent CSMA with Collision Detection

We consider a slotted p-persistent CSMA protocol with collision detection and a finite population. Here the assumptions and parameters are the same as in the previous section except that the duration of an unsuccessful transmission is now given by $b + a$, where $a \leqslant b \leqslant 1$. The parameter b stands for the effect of collision detection (we assume that b/a is an integer) such that b/a slots are necessary to abort transmission after detecting the collision. Our treatment follows that of [7] for an infinite population model of 1-persistent CSMA.

The channel throughput S is still expressed as in (1) where $\bar{I}$ is given by (7). To find $\bar{B}$ and $\bar{U}$, let us denote by $B(X)$ the mean duration of the busy period following the packet-accumulation time of X slots. Similarly, we denote by $U(X)$ the mean useful transmission time during the same busy period. Since a busy period is induced by those packets which have arrived in the preceding slot, we have

$$\bar{B} = B(1); \quad \bar{U} = U(1). \tag{29}$$

It follows, from (1) and (7), that

$$S = \frac{U(1)}{B(1) + a/[1 - (1 - g)^M]} \tag{30}$$

where $U(1)$ and $B(1)$ are determined below.

Since the duration of a successful transmission $(1 + (1/a)$ slots) is different from that of an unsuccessful transmission $(1 + (b/a)$ slots), the distribution of $N_0^{(j)}$, $j \geqslant 2$ depends on whether $T^{(j-1)}$ is successful or not. From the recursive consideration similar to [7], we have

$$\begin{aligned} B(X) = r(X) &+ \{1 + a + [1 - (1 - g)^{(1+(1/a))M}] \\ &\cdot B(1 + (1/a))\}u(X) + \{b + a + [1 - (1 - g)^{(1+(b/a))M}] \\ &\cdot B(1 + (b/a))\}[1 - u(X)] \end{aligned} \tag{31}$$

$$\begin{aligned} U(X) = \{1 &+ [1 - (1 - g)^{(1+(1/a))M}]\, U(1 + (1/a))\}u(X) \\ &+ [1 - (1 - g)^{(1+(b/a))M}]\, U(1 + (b/a))[1 - u(X)] \end{aligned} \tag{32}$$

where $r(X)$ and $u(X)$ are defined by (14) and (19), respectively. We may use (21)–(24) in the special cases stated there with obvious limiting forms in the coefficients in (31) and (32). Writing (31) for $X = 1 + (1/a)$ and $X = 1 + (b/a)$, we obtain two equations in the two unknowns, $B(1 + (1/a))$ and $B(1 + (b/a))$. Solving for them, and using them in (31), we can calculate $B(1)$. Similarly, $U(1)$ is obtained from (32). Thus, S is found by (30). We note that, in the 1-persistent case ($p = 1$), we have (24), which reduces (31) and (32) to the same form as in [7].

Figs. 3 and 4 display the throughput values of slotted p-per-

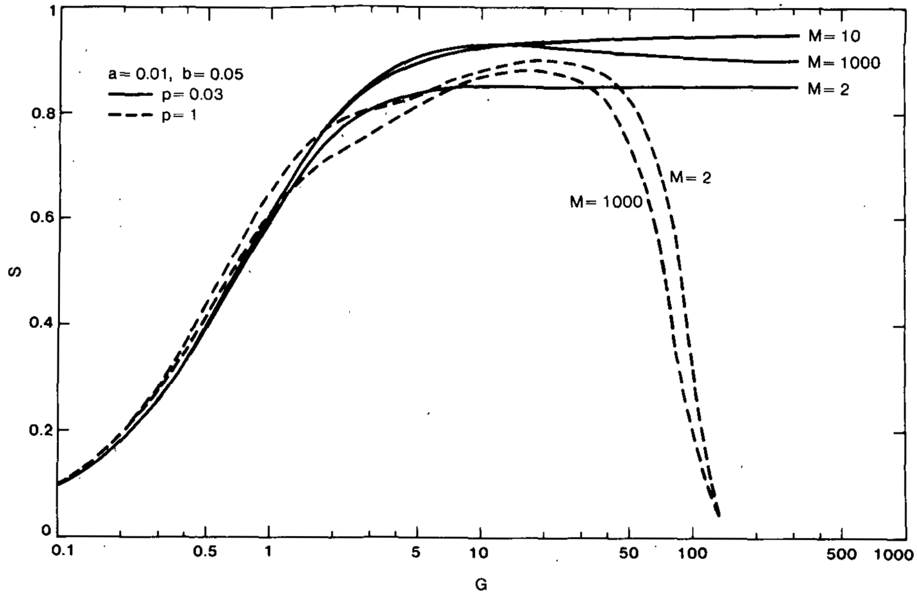

Fig. 3. Throughput of slotted p-persistent CSMA with collision detection.

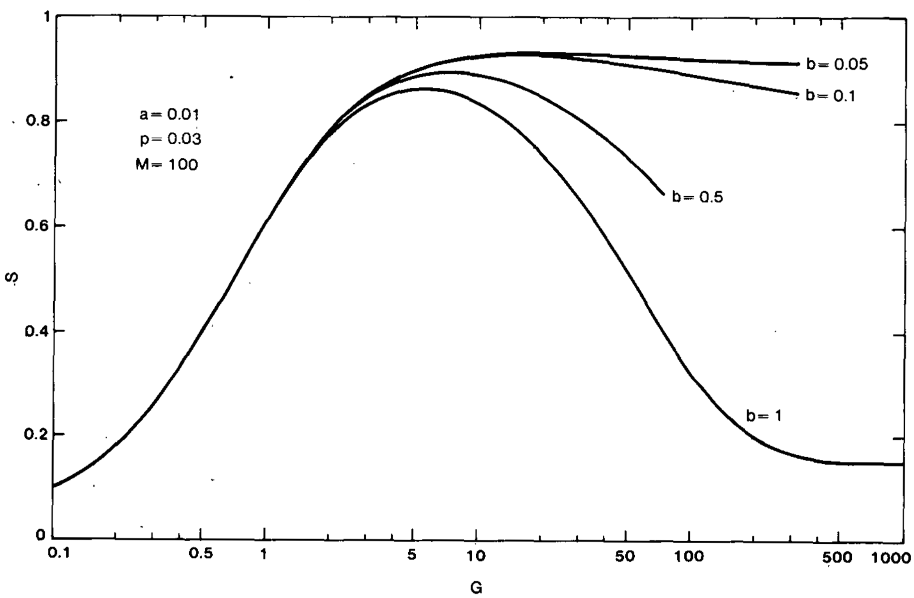

Fig. 4. Throughput of slotted 0.03-persistent CSMA with collision detection.

sistent CSMA with collision detection in terms of the total offered traffic value $G = gM/a$ ($a = 0.01$). In Fig. 3, we again observe little dependence of the maximum throughput on the number of users. Also, in 1-persistent CSMA, the throughput degrades rapidly as G exceeds some critical value. Fig. 4 demonstrates the throughput values for different values of the collision detection parameter b in the case of $M = 100$ users for 0.03-persistent CSMA. A good collision detection (small b) is seen to contribute to sustaining throughput when G is large.

IV. Unslotted Persistent CSMA

We now proceed to study unslotted persistent CSMA. Here the unit of time is still the constant packet transmission time, and a denotes the signal propagation delay (in that time unit) so that all users recognize what happened in the system a time units before. Let M be the number of users (we consider only the case of identical users), and let the time until a packet arrives at each of the empty users be independent and exponentially distributed with mean $1/g$. If a packet arrives at a user when the channel is sensed idle, he schedules start of transmission a random amount of time later. We assume that this random time is exponentially distributed with mean $1/p$, where $0 < p < \infty$. In time a after any transmission has started, it is recognized by all users, who then discard old packets (if any) and make room for new arrivals. If a packet arrives at a user when the channel is sensed busy, the start of transmission is also scheduled at an exponentially distributed time (with mean $1/p$) after the end of the transmission period. An attempted transmission is successful if it is started by breaking the idle channel, and if no other transmissions take place within a time a after the start. We call this protocol unslotted p-persistent CSMA. (The case $p = \infty$ corresponds to 1-persistent CSMA and is treated separately in Section V below. Note that the meaning of parameters g and p in unslotted CSMA is different from that in slotted CSMA.)

The channel throughput of this system can be analyzed similarly to slotted persistent CSMA. Referring to Fig. 5(a), let B and I be the durations of system busy and idle periods, respec-

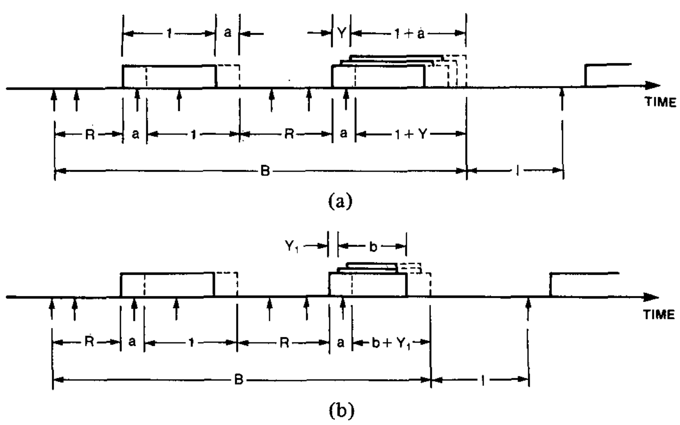

Fig. 5. Channel state in unslotted persistent CSMA. (a) Unslotted persistent CSMA. (b) Unslotted persistent CSMA with collision detection.

tively, as before; that is, the idle period is defined as the time in which the channel is idle and no packets are awaiting transmission. Upon the arrival of a packet, the system enters the busy period, which terminates at the end of a transmission period in which no packets have arrived. If U is the number of successful transmissions achieved in a busy period, then, by the renewal argument, (1) again gives the channel throughput.

A channel busy period is divided into a number of successive sub(busy)periods, each consisting of a transmission delay (denoted by R) followed by a transmission time (of duration $1 + a + Y$, where Y is defined shortly). The transmission delay is the time during which the channel is idle but with some packets awaiting transmission. If a transmission is successful, its duration is $1 + a$ ($Y = 0$); the duration of an unsuccessful transmission period is $1 + a + Y$, where Y is the transmission start time of the last colliding packet. Note that the jth subperiod, $j \geqslant 2$, is generated by the packets which arrive during $1 + Y$ in the $(j - 1)$st subperiod, whereas the first subperiod is always generated by one packet. Let B_n be the mean duration of the busy period initiated by n packets, and let U_n be the mean number of successful transmissions in the same busy period. Then, clearly,

$$\bar{B} = B_1; \quad \bar{U} = U_1. \tag{33}$$

Since $\bar{I} = 1/(gM)$, (1) gives

$$S = \frac{U_1}{B_1 + 1/(gM)}. \tag{34}$$

In the remainder of this section, we derive a system of linear equations for $\{B_n; n = 1, 2, \cdots, M\}$ and $\{U_n; n = 1, 2, \cdots, M\}$.

Let us focus our attention on a sub(busy)period which begins with n packets. Taking the origin of the time at the start of this subperiod, let $N(x)$ be the number of packets present in the system (i.e., the number of nonempty users) at time x. We first consider the distribution of R on the condition that $N(0) = n$. Note that, during the transmission delay R, each user behaves independently of others. Each of n busy users schedules his transmission after time x with probability e^{-px}. Consider the event $\{R > x, N(x) = n + m \mid N(0) = n\}$. For this even to occur, each of $M - n - m$ users does not have an arrival before x with probability e^{-gx}. For each of m users, the time until arrival plus start of transmission is less than x, with probability

$$\int_0^x ge^{-gz} \cdot e^{-p(x-z)}\, dz = \frac{g}{p - g}\,(e^{-gx} - e^{-px}). \tag{35}$$

Thus, we have

$$\text{Prob}\,[R > x, N(x) = n + m \mid N(0) = n] = e^{-pxn} \binom{M-n}{m} e^{-gx(M-n-m)} \left[\frac{g(e^{-gx} - e^{-px})}{p - g}\right]^m$$

$$x \geqslant 0 \quad m = 0, 1, 2, \cdots, M - n. \tag{36}$$

Adding (36) over all m, the number of arrivals during R, we have

$$\text{Prob}\,[R > x \mid N(0) = n] = e^{-pxn} \left(\frac{pe^{-gx} - ge^{-gx}}{p - g}\right)^{M-n} \quad x \geqslant 0 \tag{37}$$

so that the mean transmission delay conditioned on $N(0) = n$ is given by

$$E[R_{(n)}] \triangleq E[R \mid N(0) = n] = \int_0^\infty e^{-pnx} \cdot \left(\frac{pe^{-gx} - ge^{-px}}{p - g}\right)^{M-n} dx \quad n = 1, 2, \cdots, M. \tag{38}$$

We next consider the behavior of $M - 1$ users in the transmission period following the transmission delay so that $R = x$ and $N(x) = n + m$. (Since $g, p < \infty$, one event, at most, happens at a time; one user begins the transmission period.) On this condition, the event $\{Y \leqslant y\}$ occurs in the following cases. Again note that, during the first a time units, each user behaves independently. Each of $n + m - 1$ nonempty users either does not start transmission before a, with probability e^{-pa}, or starts transmission before y with probability $1 - e^{-py}$. There are three cases of behavior for each of the $M - n - m$ users who were empty at the end of R: i) no arrival during a, with probability e^{-ga}; ii) arrival before a, but transmission after a, with probability $g(e^{-ga} - e^{-pa})/(p - g)$ [similar to (35)]; and iii) arrival and transmission before y, with probability

$$\int_0^y ge^{-gz}[1 - e^{-p(y-z)}]\, dz = 1 - \frac{pe^{-gy} - ge^{-py}}{p - g}. \tag{39}$$

Therefore, we have

$$\text{Prob}\,[Y \leqslant y \mid R = x, N(x) = n + m, N(0) = n]$$

$$= (1 - e^{-py} + e^{-pa})^{n+m-1} \left[1 - \frac{pe^{-gy} - ge^{-py}}{p - g} + e^{-ga} + \frac{g(e^{-ga} - e^{-pa})}{p - g}\right]^{M-n-m}$$

$$= (1 - e^{-py} + e^{-pa})^{n+m-1} \cdot \left[\frac{p(1 - e^{-gy} + e^{-ga}) - g(1 - e^{-py} + e^{-pa})}{p - g}\right]^{M-n-m}$$

$$0 \leqslant y \leqslant a. \tag{40}$$

Note that

$$\text{Prob}\ [Y=0 \mid R=x, N(x)=n+m, N(0)=n] = e^{-pa(n+m-1)}\left(\frac{pe^{-ga}-ge^{-pa}}{p-g}\right)^{M-n-m} \tag{41}$$

is the probability of a successful transmission. Unconditioning (40) on $N(R)$ and R, using (36) and (37) successively, gives

$$f(y;n) \triangleq \text{Prob}\ [Y \leq y \mid N(0)=n] = (1-e^{-py}+e^{-pa})^{n-1} \cdot \left[\frac{p(1-e^{-gy}+e^{-ga})-g(1-e^{-py}+e^{-pa})}{p-g}\right]^{M-n} \qquad 0 \leq y \leq a \tag{42}$$

and similar unconditioning of (41) yields

$$\gamma_{(n)} \triangleq f(0;n) = e^{-pa(n-1)}\left(\frac{pe^{-ga}-ge^{-pa}}{p-g}\right)^{M-n} \qquad n=1,2,\cdots,M. \tag{43}$$

Note that $\gamma_{(n)}$ is the probability of success in the subperiod begun with n packets. The mean of Y in the similar subperiod is given by

$$E[Y_{(n)}] \triangleq E[Y \mid N(0)=n] = a - \int_0^a f(y;n)\,dy \qquad n=1,2,\cdots,M. \tag{44}$$

Finally, we consider the condition that we have k accumulated packets at the end of the transmission period. If the duration of the transmission period is $1+a+y$, those packets which arrive during $1+y$ are buffered. So, the probability of having k packets in the transmission period of duration $1+a+y$ is given by $g_k(1+y)$, where

$$g_k(y) \triangleq \binom{M}{k}(1-e^{-gy})^k e^{-gy(M-k)} \qquad k=1,2,\cdots,M. \tag{45}$$

By similar unconditioning, we have the probability p_{nk} that the next subperiod begins with k packets:

$$p_{nk} = g_k(1)f(0;n) + \int_0^a g_k(1+y)\,d_y f(y;n) \tag{46}$$

where $f(y;n)$ is given by (42). (We have assumed here that $g_k(1+y)$ is independent of n, the number of nonempty users at the beginning of the transmission period. This is based on the same assumption as that stated after (9).)

Now we are in a position to write down a system of equations for $\{B_n\}$ and $\{U_n\}$. By renewal considerations, they are given by

$$B_n = E[R_{(n)}] + 1 + a + E[Y_{(n)}] + \sum_{k=1}^{M} B_k p_{nk} \qquad n=1,2,\cdots,M \tag{47}$$

and

$$U_n = \gamma_{(n)} + \sum_{k=1}^{M} U_k p_{nk} \qquad n=1,2,\cdots,M \tag{48}$$

where $E[R_{(n)}]$, $E[Y_{(n)}]$, $\gamma_{(n)}$, and p_{nk} are given by (38), (44), (43), and (46), respectively. Thus, all we have to do to compute the throughput is to substitute the solutions of B_1 and U_1 to (47) and (48) into (34).

In Fig. 6, we plot the throughput of unslotted p-persistent CSMA for $a = 0.01$ and $M = 10$ in terms of the total offered traffic rate $G = gM$. (Note that here the scale of G is different from Figs. 2-4 and 7.) For small p, a long transmission delay R suppresses the start of actual transmission too much for small G, thus causing a reduced throughput. When p is increased, so is the probability of collision. The maximum throughput is achieved on the balance of idle period, transmission delay, and probability of collision. The smaller p is, the larger the optimal G is.

V. Unslotted 1-Persistent CSMA

In the case of 1-persistent CSMA ($p = \infty$), all the packets accumulated by the end of a transmission period are started immediately at the beginning of the next subperiod. Therefore, the duration of transmission delay R is always zero, and the fact used in Section IV that each transmission period is initiated by one user is not valid. However, a similar analysis is possible, and the channel throughput can still be calculated by using the solutions to (47) and (48), where the following replacement is made:

$$E[R_{(n)}] = 0; \quad \gamma_{(n)} = \delta_{n,1} e^{-ga(M-1)} \tag{49}$$

($\delta_{n,1} = 1$ if $n = 1$, and $= 0$ otherwise)

$$E[Y_{(n)}] = a - \int_0^a (1-e^{-gy}+e^{-ga})^{M-n}\,dy \tag{50}$$

$$p_{nk} = g_k(1)e^{-ga(M-n)} + (M-n)g\int_0^a g_k(1+y) \cdot e^{-gy}(1-e^{-gy}+e^{-ga})^{M-n-1}\,dy. \tag{51}$$

In the limit $M \to \infty$ with $G = gM$ held at a finite value, $E[Y_{(n)}]$ and p_{nk} become independent of n as

$$\bar{Y} = a - \frac{1-e^{-aG}}{G} \tag{52}$$

$$p_k = e^{-G(1+a)}\left\{\frac{G^{k+1}}{(k+1)!}[(1+a)^{k+1}-1] + \frac{G^k}{k!}\right\} \qquad k=0,1,2,\cdots. \tag{53}$$

Thus, we have

$$\bar{B} = \frac{1+a+\bar{Y}}{p_0}; \quad \bar{I} = \frac{1}{G}; \quad \bar{U} = e^{-aG} + \frac{p_1}{p_0}e^{-aG}. \tag{54}$$

Then we recover the result in [6] for an infinite population model:

$$S = \frac{Ge^{-G(1+2a)}[1+G+aG(1+G+aG/2)]}{G(1+2a)-(1-e^{-aG})+(1+aG)e^{-G(1+a)}}. \tag{55}$$

The values of S in (55) when $a = 0.01$ are plotted in Fig. 6 with a label $p = \infty$.

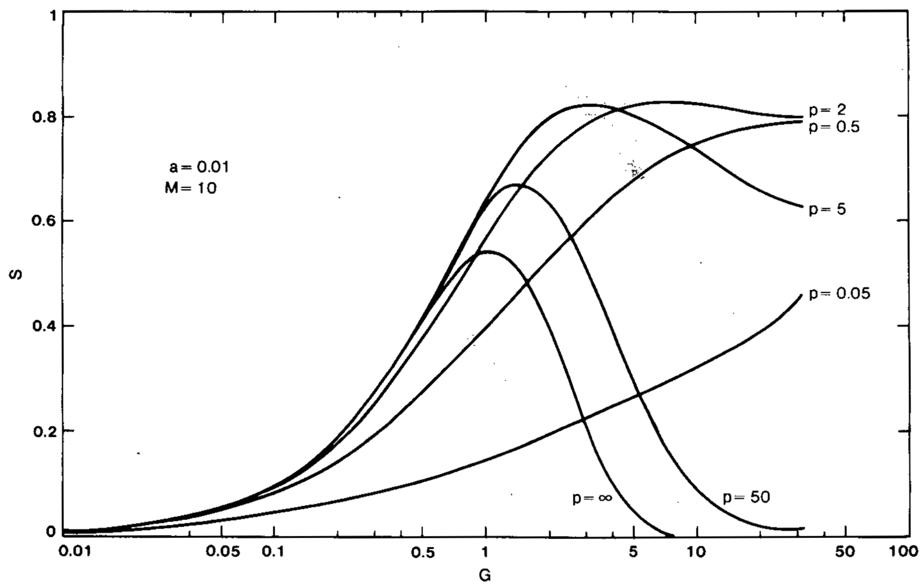

Fig. 6. Throughput of unslotted p-persistent CSMA.

VI. Unslotted Persistent CSMA with Collison Detection

CSMA with collision detection can be treated similarly. Let the collision detection in an unslotted system be such that the duration of an unsuccessful transmission $b + a + Y_1$, where Y_1 is the transmission start time of the *first* colliding packet. (Unlike CSMA without collision detection, note here that it is the first colliding packet that stops the transmission of the leading packet, which lingers till the last moment.) Note that all but the user who initiates a transmission period (called a leading user) know the transmission start at time a. So, they can stop transmissions at time $a + b$ (if started). However, a leading user knows the transmission start by the first colliding user at time Y_1 and stops transmission at time $Y_1 + a + b$. The distribution of transmission delay R and the probability of success in a subperiod, given that $N(0) = n$, are the same as before; they are given by (36)–(38) and (43), respectively. An illustration of channel state is given in Fig. 5(b).

Let us find the probability of the event $\{Y_1 > y\}$ conditioned on the event $\{R = x, N(x) = n + m, N(0) = n\}$ and that the transmission is unsuccessful. This event occurs when each of $n + m - 1$ nonempty users does not start transmission before y, with probability $e^{-py(n+m-1)}$, and when the time of transmission start following an arrival at each of $M - n - m$ empty users is after y, with probability $(pe^{-gy} - ge^{-py})/(p - g)$ [calculated similarly to (39)]. Since each user behaves independently during $0 \leqslant y \leqslant a$, we have

$$\text{Prob}\,[Y_1 > y \mid \text{collision}, R = x, N(x) = n + m, N(0) = n]$$

$$= \frac{e^{-py(n+m-1)}\left(\dfrac{pe^{-gy} - ge^{-py}}{p - g}\right)^{M-n-m} - e^{-pa(n+m-1)}\left(\dfrac{pe^{-ga} - ge^{-pa}}{p - g}\right)^{M-n-m}}{1 - e^{-pa(n+m-1)}\left(\dfrac{pe^{-ga} - ge^{-pa}}{p - g}\right)^{M-n-m}} \tag{56}$$

Unconditioning (56) on R and $N(R)$ [by using (36)] and taking the average, we get

$$E[Y_{1(n)}] \triangleq E[Y_1 \mid \text{collision}, N(0) = n]$$

$$= \frac{\displaystyle\int_0^a f_1(y; n)\, dy - a\gamma_{(n)}}{1 - \gamma_{(n)}} \tag{57}$$

where $\gamma_{(n)}$ is given by (43), and

$$f_1(y; n) \triangleq \text{Prob}\,[Y_1 > y \mid N(0) = n]$$

$$= e^{-py(n-1)}\left(\frac{pe^{-gy} - ge^{-py}}{p - g}\right)^{M-n}. \tag{58}$$

Note also $\gamma_{(n)} = f_1(a; n)$.

The probability that a successful subperiod begun with n nonempty users ends up with k nonempty users is obviously given by

$$p_{nk}^{(s)} = g_k(1) \tag{59}$$

where $g_k(y)$ is defined in (45). In the case of an unsuccessful transmission, since the packets are accumulated over the duration $b + Y_1$, the corresponding probability is given by

$$p_{nk}^{(c)} = \frac{-1}{1 - \gamma_{(n)}} \int_0^a g_k(b + y) d_y f_1(y; n). \tag{60}$$

We are now able to write a system of equations for $\{B_n\}$. By renewal consideration, as in Section III, we have

$$B_n = E[R_{(n)}] + \gamma_{(n)}\left\{1 + a + \sum_{k=1}^{M} B_k p_{nk}{}^{(s)}\right\}$$

$$+ [1 - \gamma_{(n)}]\left\{b + a + E[Y_{1(n)}] + \sum_{k=1}^{M} B_k p_{nk}{}^{(c)}\right\} \tag{61}$$

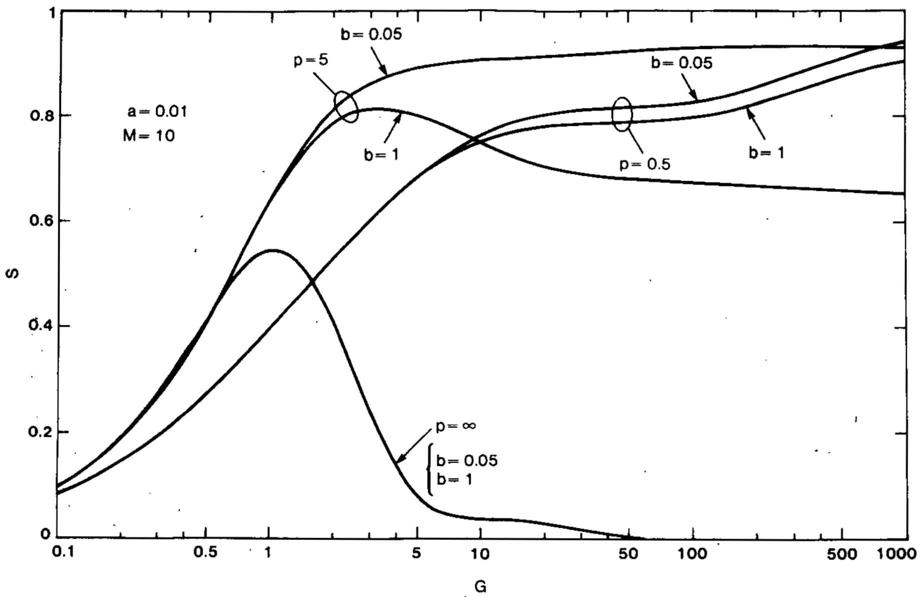

Fig. 7. Throughput of unslotted persistent CSMA with collision detection.

which may be rewritten as

$$B_n = E[R_{(n)}] + \gamma_{(n)} + [1 - \gamma_{(n)}](b + a) + \int_0^a f_1(y; n)\, dy + \sum_{k=1}^{M} p_{nk} B_k \qquad n = 1, 2, \cdots, M \tag{62}$$

where

$$p_{nk} \triangleq \gamma_{(n)} p_{nk}{}^{(s)} + [1 - \gamma_{(n)}] p_{nk}{}^{(c)} = \gamma_{(n)} g_k(1) - \int_0^a g_k(b + y)\, d_y f_1(y; n) \tag{63}$$

is the probability that the next subperiod begins with k packets. Similarly, for $\{U_n\}$ we have

$$U_n = \gamma_{(n)} + \sum_{k=1}^{M} p_{nk} U_k \qquad n = 1, 2, \cdots, M. \tag{64}$$

Thus, by solving (62) for $\{B_n\}$ and (64) for $\{U_n\}$, we can find B_1 and U_1, which are to be used in (34) to compute S.

Fig. 7 shows the throughput of unslotted p-persistent CSMA with collision detection for some combinations of p and b in the case of $a = 0.01$ and $M = 10$. Here the throughput values appear to depend little on b, particularly for large values of p, unless b is close to a. The reason for this is explained below by using the explicit expressions for $\bar{B}$ and $\bar{U}$ for an infinite population model of 1-persistent CSMA.

In unslotted 1-persistent CSMA with collision detection, the system of equations for $\{B_n\}$ and $\{U_n\}$ is given by

$$B_n = e^{-ga(M-n)} + [1 - e^{-ga(M-n)}] \cdot \left[b + a + \frac{1}{g(M-n)} \right] + \sum_{k=1}^{M} p_{nk} B_k \tag{65}$$

$$U_n = \delta_{n,1} e^{-ga(M-1)} + \sum_{k=1}^{M} p_{nk} U_k \qquad n = 1, 2, \cdots, M \tag{66}$$

where

$$p_{nk} = g_k(1) e^{-ga(M-n)} + (M - n) g \int_0^a g_k(b + y) \cdot e^{-gy(M-n)}\, dy. \tag{67}$$

In the limit $M \to \infty$ with $G = gM$ fixed at a finite value, p_{nk} becomes independent of n, and we obtain explicitly

$$p_0 = e^{-G(1+a)} + e^{-bG}(1 - e^{-2aG})/2 \tag{68}$$

$$p_1 = Ge^{-G(1+a)} + Ge^{-bG} \left[\left(\frac{b}{2} + \frac{1}{4G} \right) (1 - e^{-2aG}) - \frac{a}{2} e^{-2aG} \right] \tag{69}$$

$$B_1 = \frac{1}{p_0} \left[e^{-aG} + (1 - e^{-aG}) \left(b + a + \frac{1}{G} \right) \right] \tag{70}$$

$$U_1 = \left(1 + \frac{p_0}{p_1} \right) e^{-aG}. \tag{71}$$

Substituting these expressions, as well as $gM = G$, into (34), we obtain

$$S = \frac{G(1 + G)e^{-G(1+2a)} + Ge^{-G(b+a)}[(bG/2+)(1 - e^{-2aG}) - aGe^{-2aG}/2]}{e^{-G(1+a)} + Ge^{-aG} + (1 - e^{-aG})[1 + (b + a)G] + e^{-bG}(1 - e^{-2aG})/2}. \tag{72}$$

Fig. 8 plots S in (72) in the case of $a = 0.01$. As in Fig. 7, S depends little on b unless b is small. This comes from the following behavior of B_1 and U_1 as G changes. When G is small, we are likely to have successful transmissions, so that the effects of collision detection are nominal. When G gets large and the collision detection becomes effective, the increase in the number of subperiods in a busy period (as e^{bG} in $1/p_0$) outweighs the decrease in the duration of each subperiod

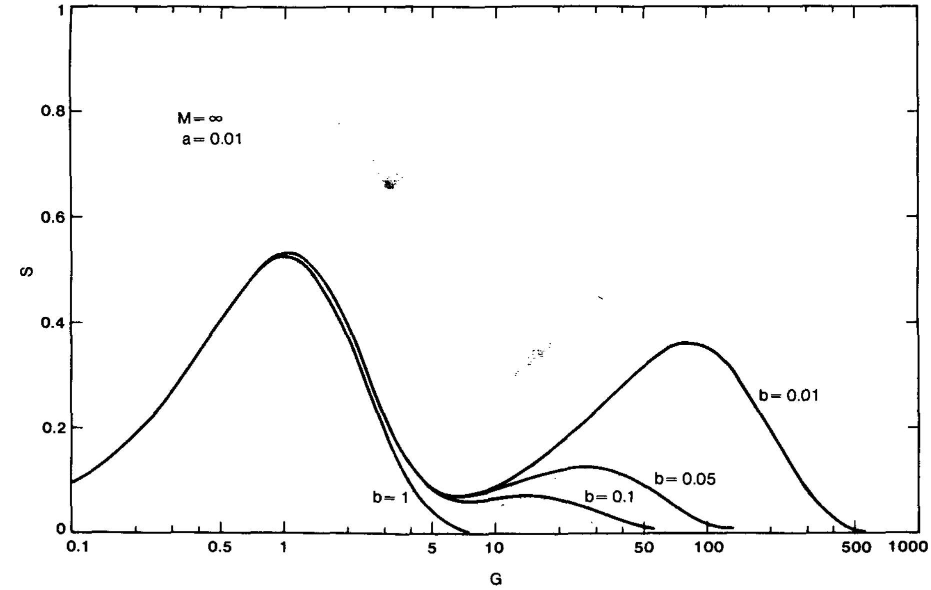

Fig. 8. Throughput of unslotted 1-persistent CSMA with collision detection.

(linear in b). As a result, the duration of the whole busy period grows rapidly (as be^{bG}), while the growth in the number of successful transmissions in a busy period levels off and then decreases due to increased collisions. Thus, the collision detection feature seems ineffective in these cases.

On the other hand, if b is very small, the double peaks in Fig. 8 are striking. As G increases, the first peak (near $G = 1$) corresponds to the point of balance between the durations of the idle period and the busy period which is most likely to contain one transmission period which is successful. The second peak (whose position and height depend on b) corresponds to the point of maximum number of successful transmissions per unit length of a busy period (the idle period has little effect here because of large G). The smaller b is, the faster the unsuccessful transmission periods are ended. This is why a smaller b brings about a larger throughput for a given value of G in this region. Furthermore, the optimal G which maximizes the throughput (in this region) is larger for smaller values of b, since more transmissions can then be started for the same duration of the busy period.

VII. Conclusion

In this paper, we have given a unified throughput analysis for slotted and unslotted persistent CSMA with and without collision detection. Due to the assumption of exponentially or geometrically distributed idle periods, the intervals between two successive epochs at which the system enters the idle period are independent and identically distributed. Therefore, the system state can be modeled as a regenerative process. Through renewal arguments, we have calculated the channel throughput for various persistent CSMA systems. We have obtained several new results [(25)–(27); (31) and (32) with (14), (19), and (21)–(23); (47) and (48); (62) and (64); (65) and (66); and (72)] and consistently rederived some known results [(28); (31) and (32) with (24); and (55)].

Two major assumptions we have adopted in this paper are i) the times until next arrival at each empty user are independent and identically (exponentially or geometrically) distributed; and ii) the number of packets accumulated at the end of a transmission period is simply the number of arrivals during that transmission period, in disregard of the packets which were already buffered at the beginning of the transmission period (they are discarded). Assumption i) is essential in making analysis tractable, in virtue of the memoryless property. Assumption ii) was used to be consistent with the previous treatment in [6] (in fact, in (28), we have derived one of the previous results as a special case). One of the advantages drawn from this assumption is that, in the analysis of slotted persistent CSMA (Section II), the subperiods $B^{(j)}$, $j \geqslant 2$, are statistically independent and identical, and this fact brings about the closed-form expression for throughput as (25). Instead of ii) above, we could have assumed that all the accumulated packets are kept in the buffer until they are successfully transmitted. Then, the number of packets accumulated during a transmission period would depend on the number of packets buffered at the beginning of the transmission period. We note that the throughput analysis of (slotted and unslotted) CSMA, based on the latter assumption, is still possible; only then would we have a system of linear equations for $\{B_n\}$ and $\{U_n\}$ (see Section IV for their definition) like (47) and (48). Thus, assumption ii) is not essential for the tractability of analysis.

Since the major purpose of the present paper is to provide the explicit formula for throughput evaluation for various finite population models, we have not gone into the area of optimization (with respect to G and p) or the comparison of optimized throughput values (capacities) among protocols. These subjects, as well as possible ramification of models [e.g., the above-mentioned alternative assumption to ii)], remain to be elaborated.

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